Supersymmetric Dualities in three dimensions

First year PhD Workshop - University of Milan

Davide Morgante - PhD Workshop 2021/2022

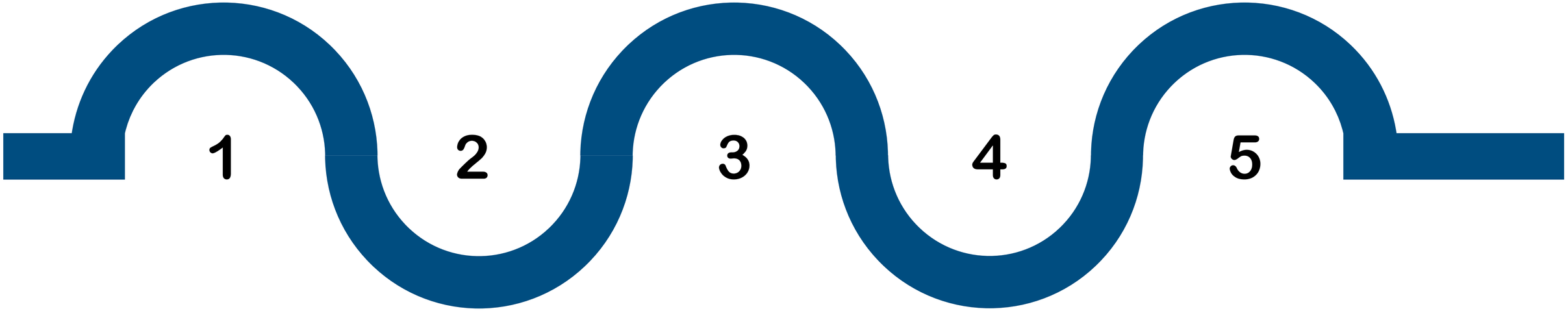
Supersymmetric Dualities in three dimensions

Question 2

What are
SUSY dualities?

Question 4

How to
check dualities?



Question 1

What is
supersymmetry?

Question 3

Why study
SUSY dualities?

Question 5

Why
three dimensions?

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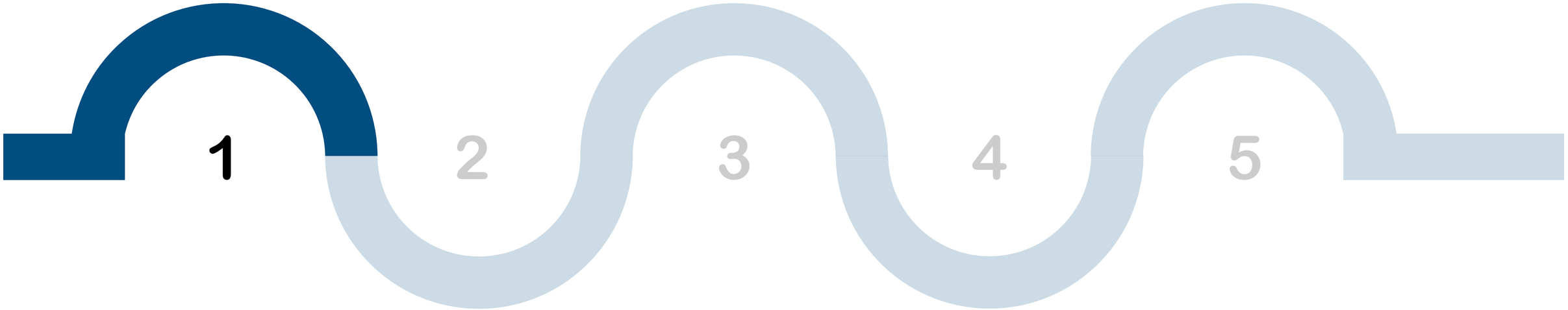

Question 1

What is supersymmetry?

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Question 1

What is supersymmetry?

- Supersymmetry (SUSY) is a space-time symmetry: only coherent extension of Poincarè algebra to include fermonic generators (anti-commutators) (Coleman-Mandula & Haag-Lopuszanski-Sohnius)

- Relationship between bosons and fermions

For us SUSY is the perfect theoretical laboratory for strongly coupled gauge dynamics

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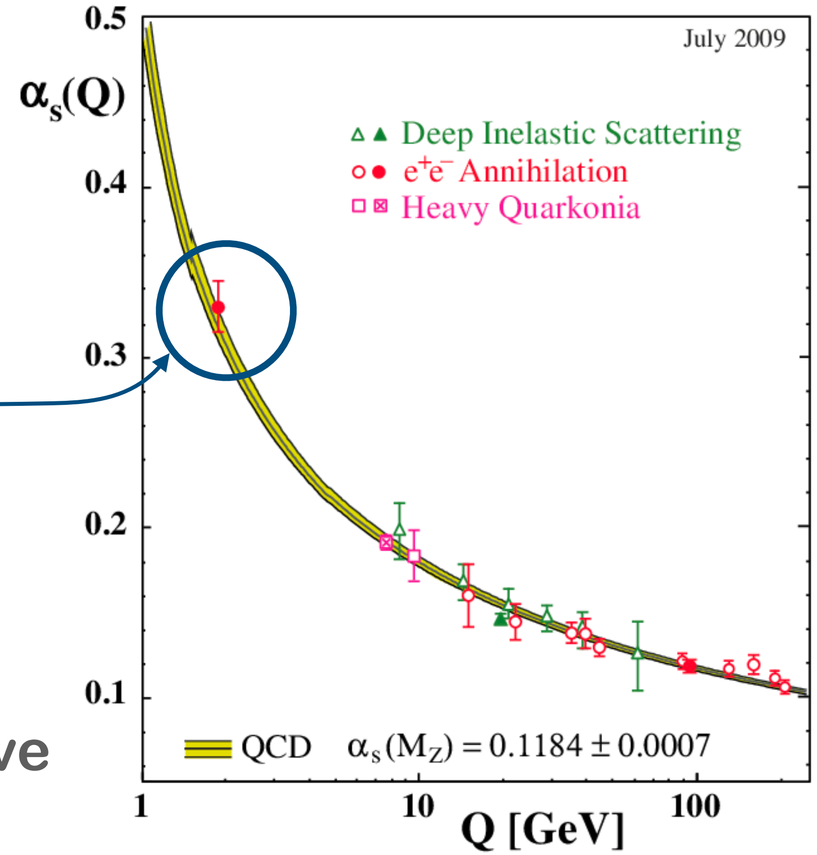
Question 1

What is supersymmetry?

Take a step back to ordinary QFT

Strong coupling dynamics, few analytical tools, no perturbative expansion



In SUSY (usually)

Holomorpy provides exact results in the non-perturbative regime

Strong/Weak dual descriptions

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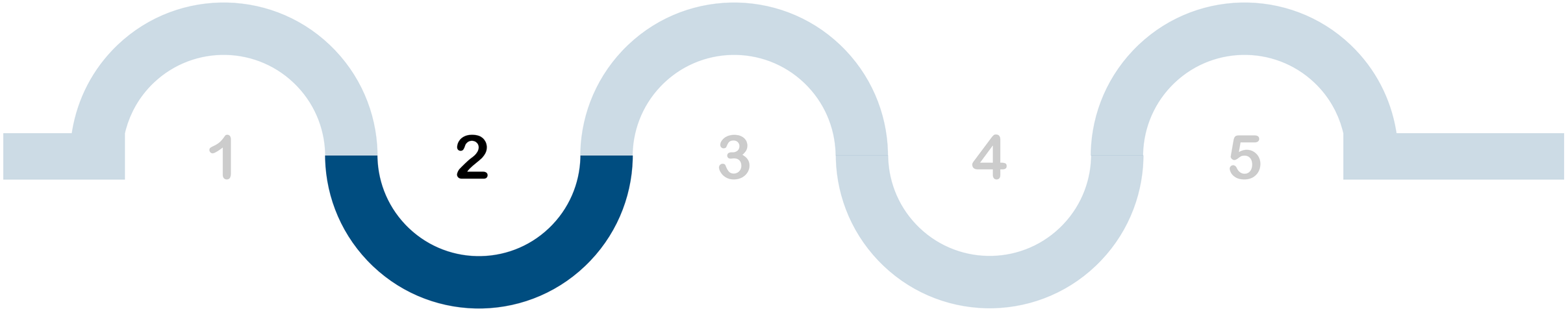

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Question 2

What are SUSY dualities?

Dualities provide different descriptions for the same physical dynamics

1994 → Seiberg provided the first example* of SUSY duality

hep-th/9411149

We demonstrate electric-magnetic duality in N=1 supersymmetric non-Abelian gauge theories in four dimensions by presenting two different gauge theories (different gauge groups and quark representations) leading to the same non-trivial long distance physics. The quarks and gluons of one theory can be interpreted as solitons (non-Abelian magnetic monopoles) of the elementary fields of the other theory. The weak coupling region of one theory is mapped to a strong coupling region of the other. When one of the theories is Higgsed by an expectation value of a squark, the other theory is confined. Massless glueballs, baryons and Abelian magnetic monopoles in the confining description are the weakly coupled elementary quarks (i.e. solitons of the confined quarks) in the dual Higgs description.

$SU(N)$ with F flavors

$\updownarrow$

$SU(F-N)$ with F flavors

*with minimal SUSY in 4d

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Question 2

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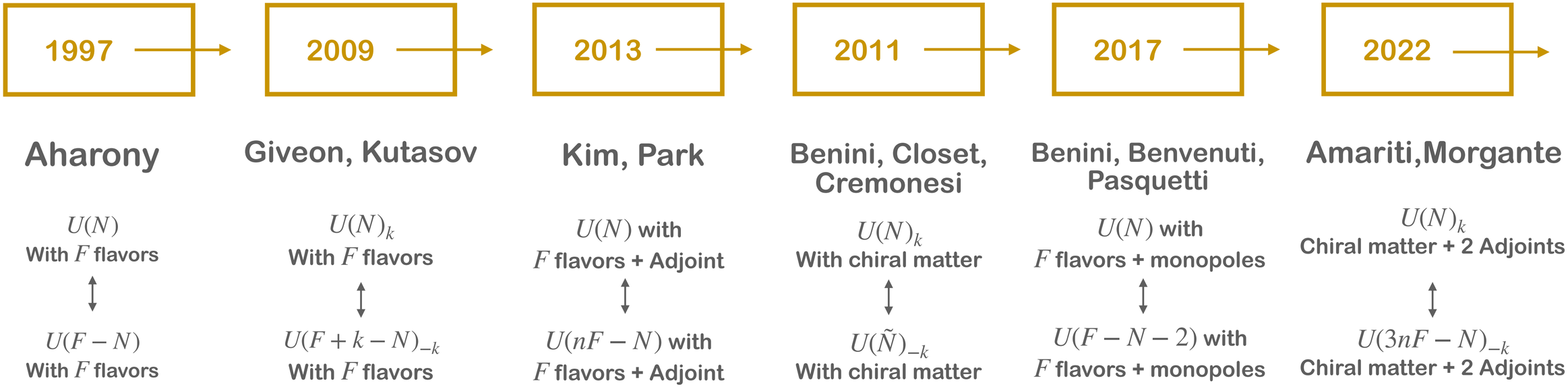
After Seiberg, many other dualities have been conjectured.

Few examples in 3d are



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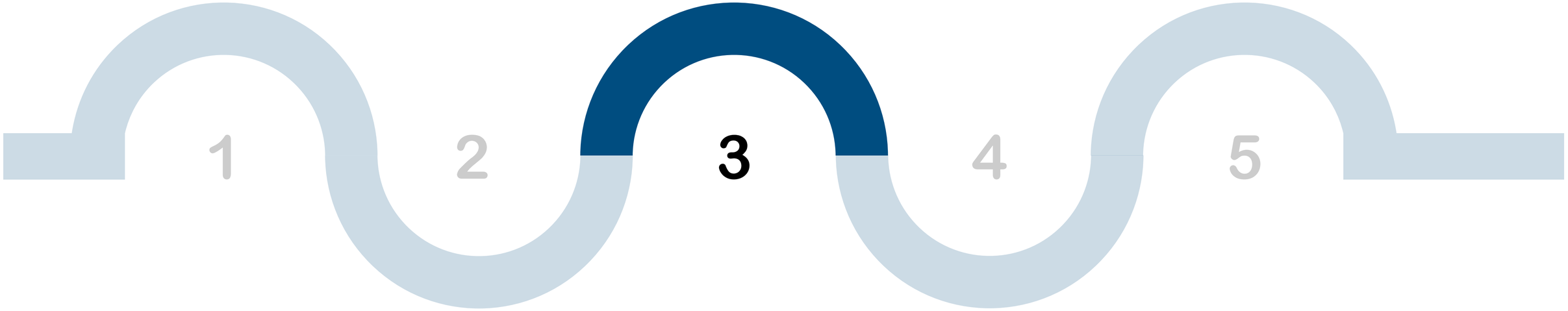

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Question 3

Why study SUSY dualities?

Some, but not all, examples are:

- Non perturbative dynamics
- Strong/Weak correspondence
- Relationship between different dimensions
- Mathematical identities

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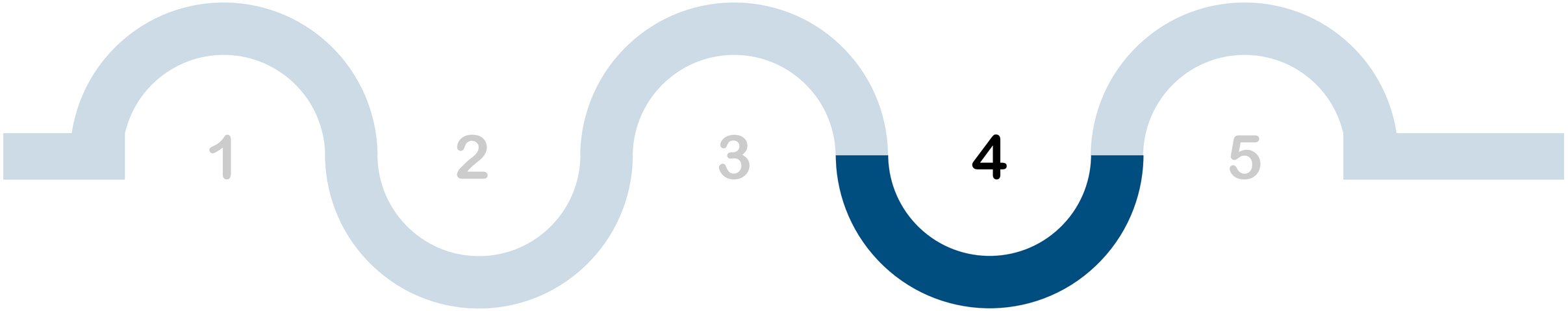

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Old way

Mapping operators
Moduli space
't Hooft anomaly matching

New way

Localization
Partition functions

Standard paradigm: information of QFT in path integral, evaluate them in a perturbative expansion. Works only at small coupling.

$$\int \mathscr{D}\phi e^{iS[\phi]/\hbar} \to \sum_n g^n \langle \phi_1 \cdots \phi_n \rangle \qquad g \ll 1$$

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Question 4

How to check dualities?

New paradigm: use SUSY and localization to get exact results on partition functions

$$\int_{S^3_b} \mathscr{D}\phi e^{iS[\phi]/\hbar} \to \frac{1}{|W|} \int \prod_{i=1}^{rank\, G} \mathrm{d}\sigma_i \; \mathscr{Z}_V(\sigma) \prod_{I \in \mathscr{R}} \mathscr{Z}_I(\sigma, \mu) \; e^{-i\pi(\lambda \,\mathrm{Tr}\, \sigma + k \,\mathrm{Tr}\, \sigma^2)}$$

From infinite dimensional integral to finite dimensional integral! This is an exact non-perturbative result!

Idea behind localization: use symmetries to reduce the dimension of the path integral, semiclassical analysis is exact!

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Question 4

How to check dualities?

Localization provides a tool to check dualities between two theories. In essence, when theory A is dual to theory B we should have that

$$\mathcal{Z}(A) = \mathcal{Z}(B)$$

Partition function theory A = Partition function theory B

Mathematicians came to the rescue: many integral equalities, as well as functional properties, were thoroughly studied before and provided the checks needed

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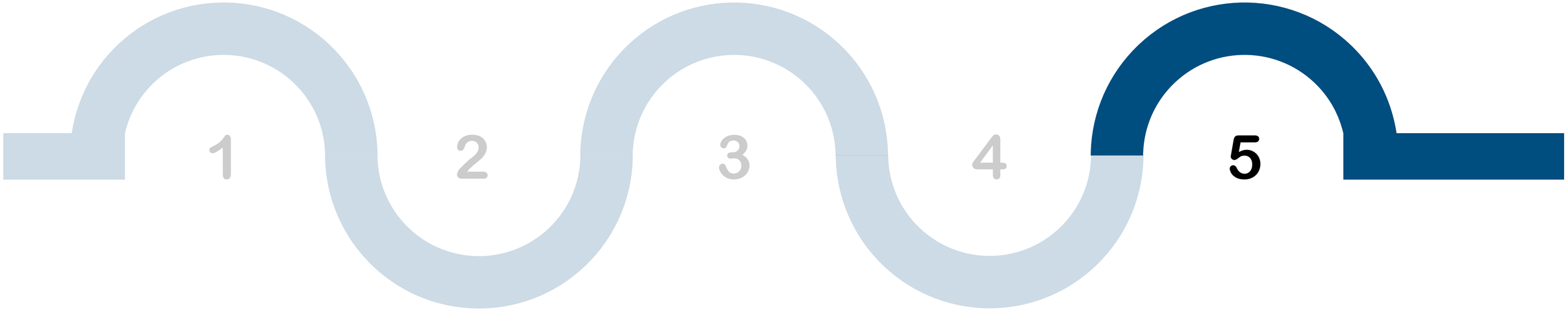

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- Three dimensions provide a useful playground for SUSY dualities. These theories are easier in many ways with respect to their 4d cousins. They are also richer given the necessary presence of monopoles and topological terms for the gauge fields.
- An example in a context other that SUSY is the use of 3d dualities in condensed matter physics.
- AdS/CFT correspondence: gravity in 4d is dual to a 3d CFT

Thank You!

Backup Slides

Supersymmetric Dualities in three dimensions

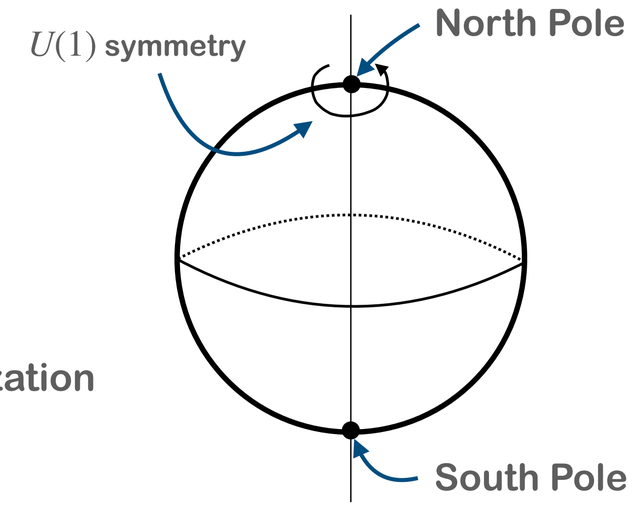
Brief overview of localization: basic example

$$\int_{S^2} e^{it\cos\theta} \mathrm{d}Vol(S^2)$$

Direct integration

$$\frac{4\pi}{t}\sin t$$

Localization



North & South Pole are $U(1)$ fixed points

$$\frac{2\pi}{it}\left(\frac{e^{itz_{NP}}}{1}+\frac{e^{itz_{SP}}}{-1}\right)=\frac{4\pi}{t}\sin t$$

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Theory: $3d\ \mathcal{N}=2\ U(N)$ **SQCD with** F **quarks and anti-quarks and one adjoint field** X

Adjoint field

$$\mathcal{Z}_{U(N)}(\tau,\mu,\nu;\omega)=\frac{\Gamma_h(\tau)^N}{N!}\int\prod_{i=0}^{N}\frac{\mathrm{d}\sigma_i}{\sqrt{-\omega_1\omega_2}}e^{i\pi\lambda\sigma_i}\prod_{1\leq i<j\leq N}\frac{\Gamma_h(\tau\pm(\sigma_i-\sigma_j))}{\Gamma_h(\pm(\sigma_i-\sigma_j))}$$
$$\times\prod_{i=1}^{N}\prod_{a=1}^{F}\Gamma_h(\mu_a+\sigma_i,\nu_a-\sigma_i)$$

Fundamentals (Quarks), Anti-Fundamentals (anti-Quarks)