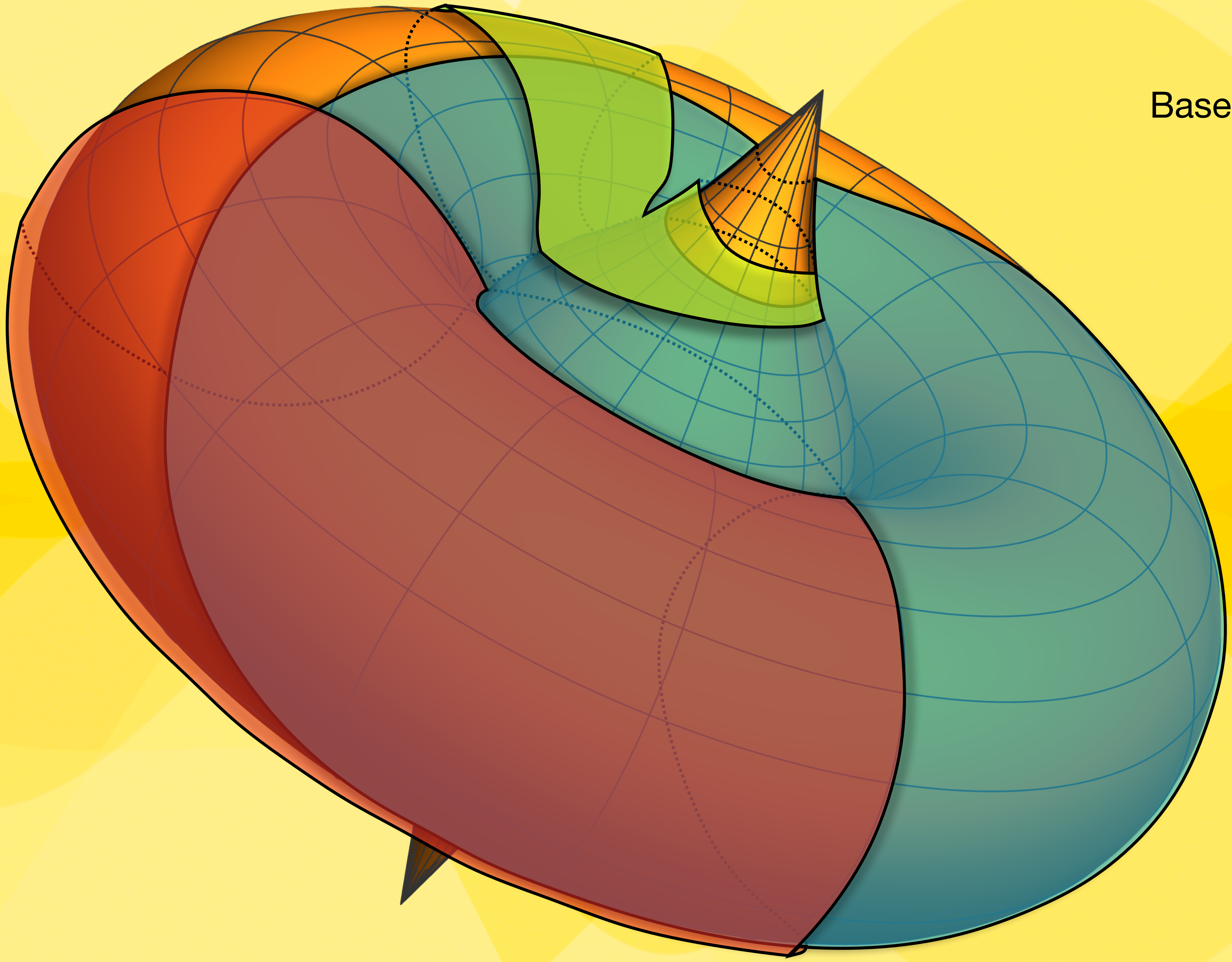
Spindly M5s

Based on 2309.11362 - A. Amariti, S. Mancani, DM, N. Petri, A. Segati



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Summary

What we did: consider the B3W $4d$ model coming from wrapping M5-branes on Riemann surface and compactify to $2d$ on a Spindle. Find the central charge of the theory to then match it to the sugra calculation

Outline

General introduction

Field Theory

The gravity dual

Outline

General introduction

- Spindle geometry $\mathbb{WCP}^1_{[n_N,n_S]}$: Twist and anti-twist
- The M5 world-volume theory: $6d\ \mathcal{N}=(2{,}0)\ A_{N-1}$ SCFT
- Wrapping M5s on Riemann surfaces and T_N blocks
- The B3W model

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Important insights into strongly coupled SCFT by realizing them as RG fixed points of compactification of higher-dimensional QFTs

Foundational work Maldacena & Nunez $4d$ SCFT from M5-branes on Riemann surface $\Sigma_g \implies$ SUSY preserved by topological twist

No covariantly constant spinor $(\partial_\mu + \omega_\mu)\epsilon = 0$, couple to background R-symmetry $A_\mu = -\omega_\mu$, then $(\partial_\mu + \omega_\mu + A_\mu)\epsilon = 0 \implies \epsilon$ constant

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General introduction

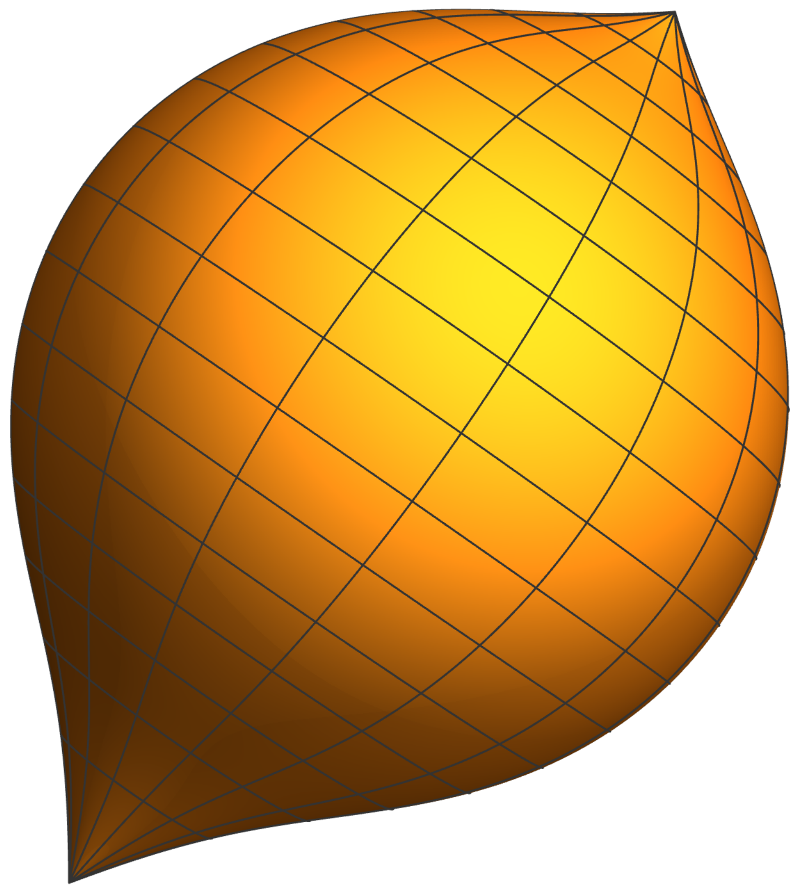
- Spindle geometry $\mathbb{WCP}^1_{[n_N,n_S]}$: Twist and anti-twist

Condition $A_\mu = -\omega_\mu$ equivalent to choosing right flux for R-symmetry background

$$\frac{1}{2\pi}\int_{\Sigma_g} F^R = 2(g-1)$$

General introduction

- Spindle geometry $\mathbb{WCP}^1_{[n_N,n_S]}$: Twist and anti-twist



More general solutions Σ is not compact manifold, but *orbifold*. The spindle is one such geometry where SUSY is preserved [Ferrero et al. ‘21, Ferrero et al. ’22, …]

Spindle: topologically S^2 with conical deficit angles at poles

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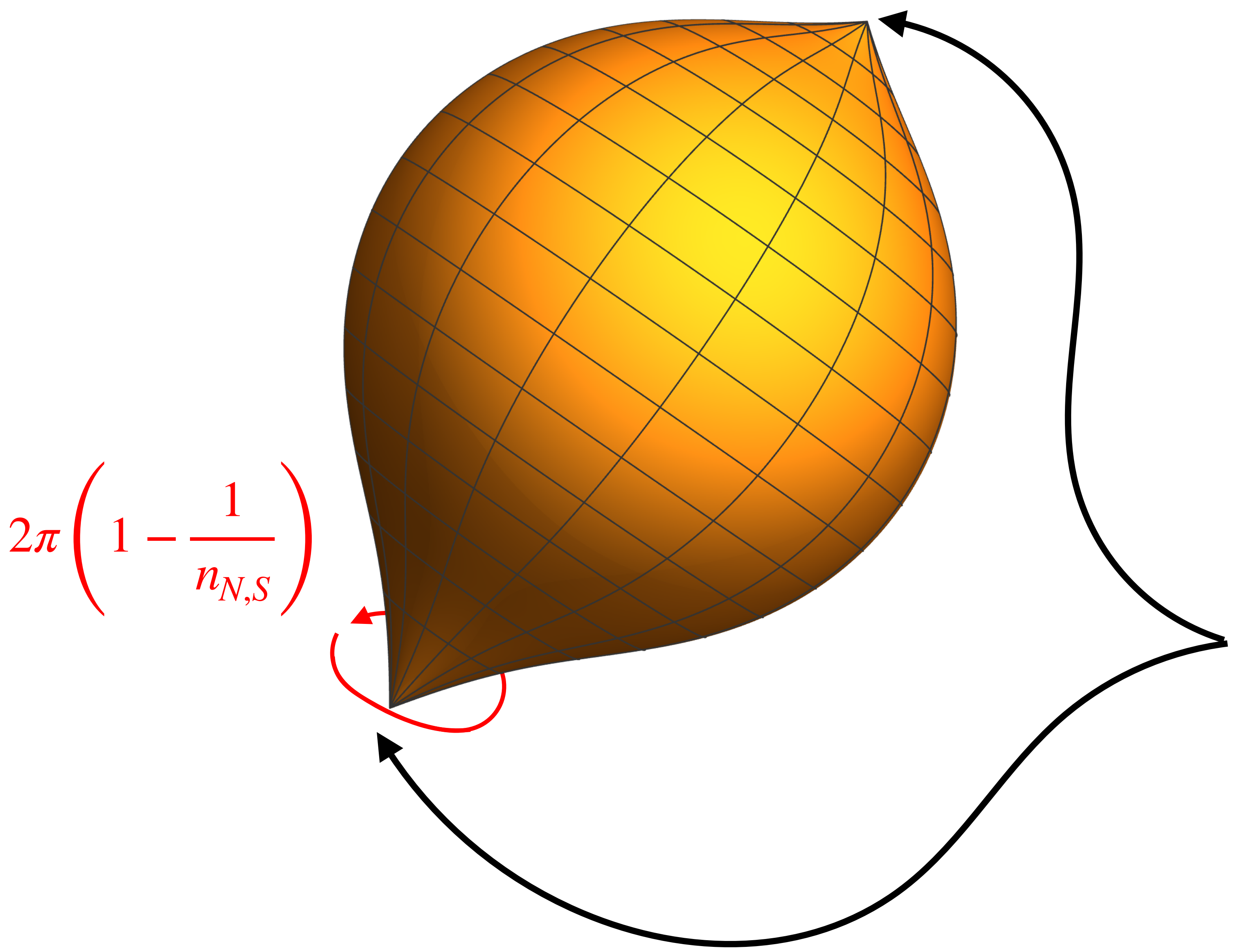

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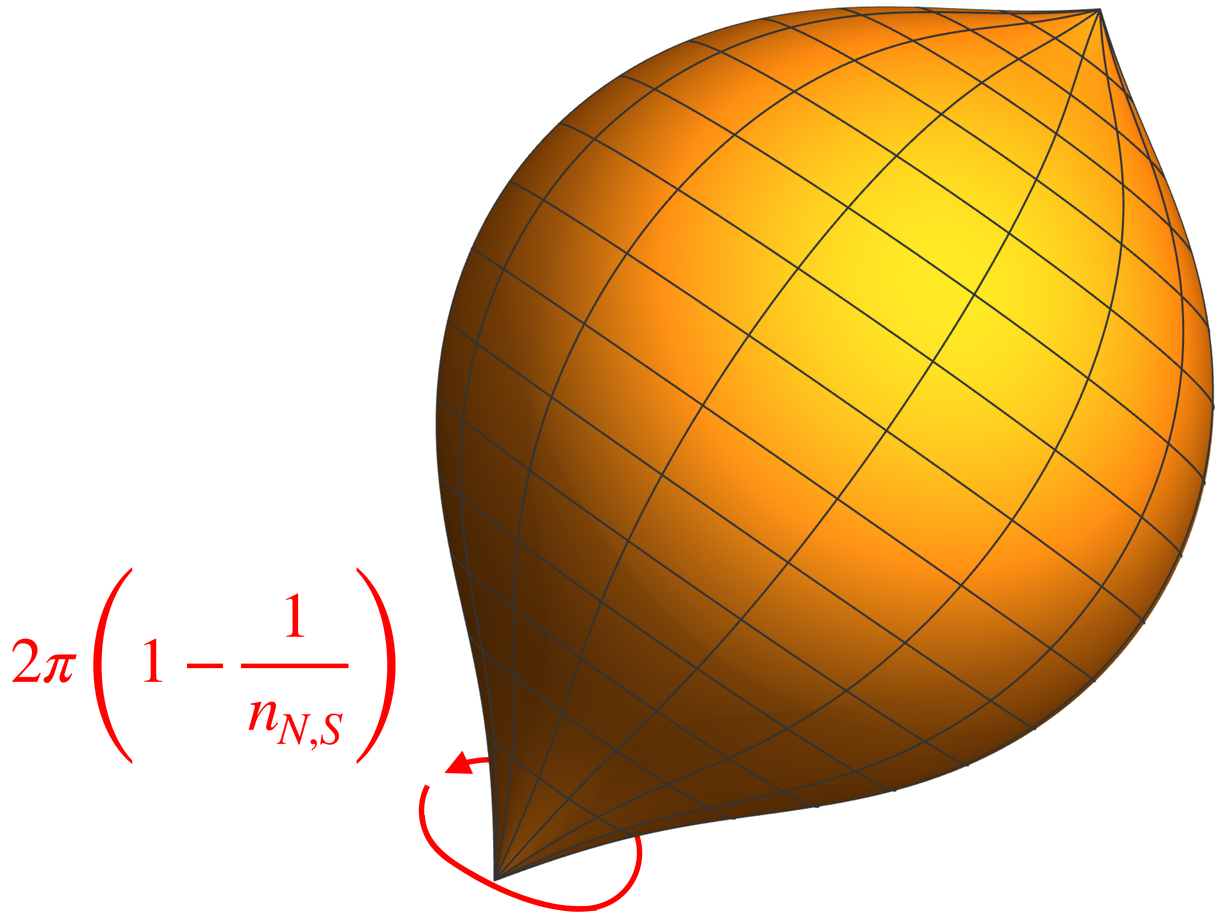

SUSY preserved also in non-trivial way

- Twist $$\frac{1}{2\pi}\int_{\mathbb{WCP}^1_{[n_N,n_S]}} F^R = \frac{n_N+n_S}{n_N n_S}$$

- Anti-twist $$\frac{1}{2\pi}\int_{\mathbb{WCP}^1_{[n_N,n_S]}} F^R = \frac{n_N-n_S}{n_N n_S}$$

General introduction

- The M5 world-volume theory: $6d\ \mathcal{N} = (2{,}0)\ A_{N-1}$ SCFT

The world-volume theory of an M5-brane is a $6d\ \mathcal{N} = (2{,}0)$ SCFT. No known lagrangian formulation

From $D = 11$ $\mathrm{SO}(5)$ normal bundle to M5 couples to R-symmetry $\mathrm{Sp}(2) \simeq \mathrm{SO}(5)$.

By stacking M5-branes we get $6d\ \mathcal{N} = (2{,}0)\ A_N$ SCFT [Strominger ’95]

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General introduction

- Wrapping M5s on Riemann surfaces and T_N blocks

Take M5 wrap on S^1 with radius $R_6 \implies 5d\ \mathcal{N}=2$ SYM

$$\int d^5x \frac{1}{g_5^2} \mathrm{tr} F \wedge \star F + \cdots \implies g_5^2 \propto R_6$$

Compactify on another S^1 with radius $R_5 \implies 4d\ \mathcal{N}=4$ SYM

$$\int dx_5 \int d^4x \frac{1}{g_5^2} \mathrm{tr} F \wedge \star F + \cdots \implies g_5^{-2} dx_5 = g_4^{-2} \implies \frac{1}{g_4^2} \sim \frac{R_5}{R_6}$$

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General introduction

- Wrapping M5s on Riemann surfaces and T_N blocks

Upshot: M5 wrapped on $T^2 \implies 4d\ \mathcal{N} = 4$ SYM w/ $g_4^{-2} \sim R_5/R_6$

Compactify in opposite order $\implies 4d\ \mathcal{N} = 4$ SYM w/ $g_4^{-2} \sim R_6/R_5$

This is S-duality!

We can generalize for any (punctured) Riemann surfaces $\Sigma_{g,n}$: class-S theories [Gaiotto '09]

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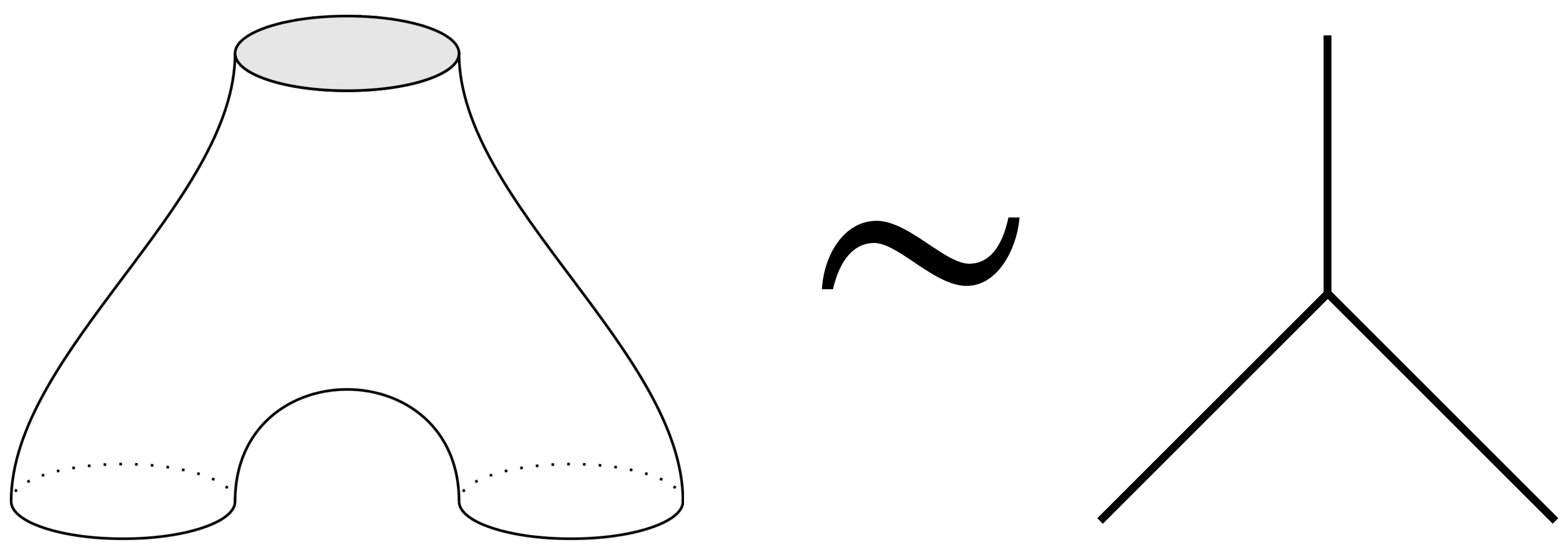
General introduction

- Wrapping M5s on Riemann surfaces and T_N blocks

Any Riemann surface can be decomposed into pair of pants

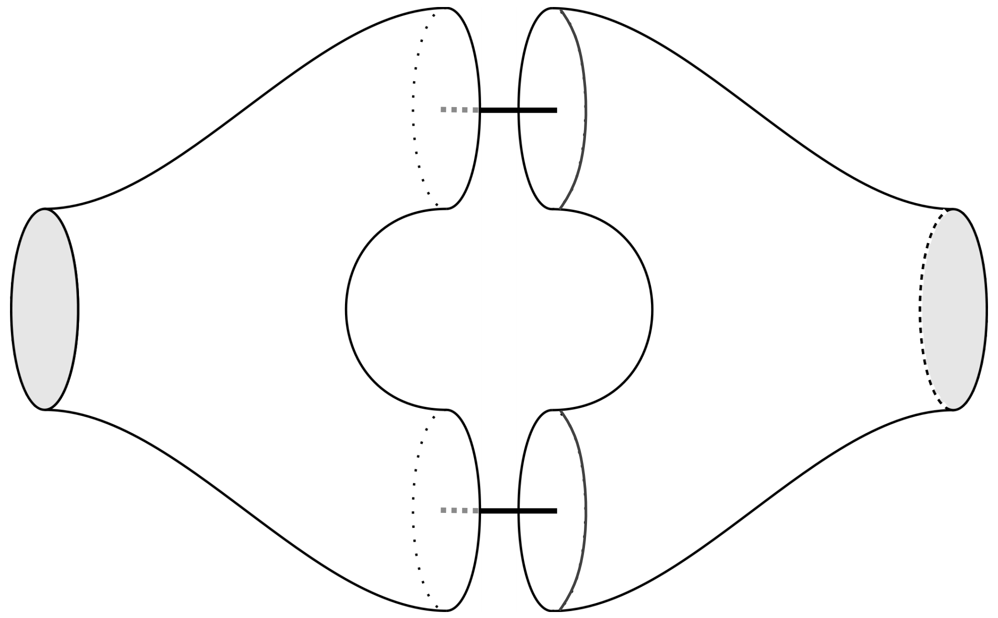


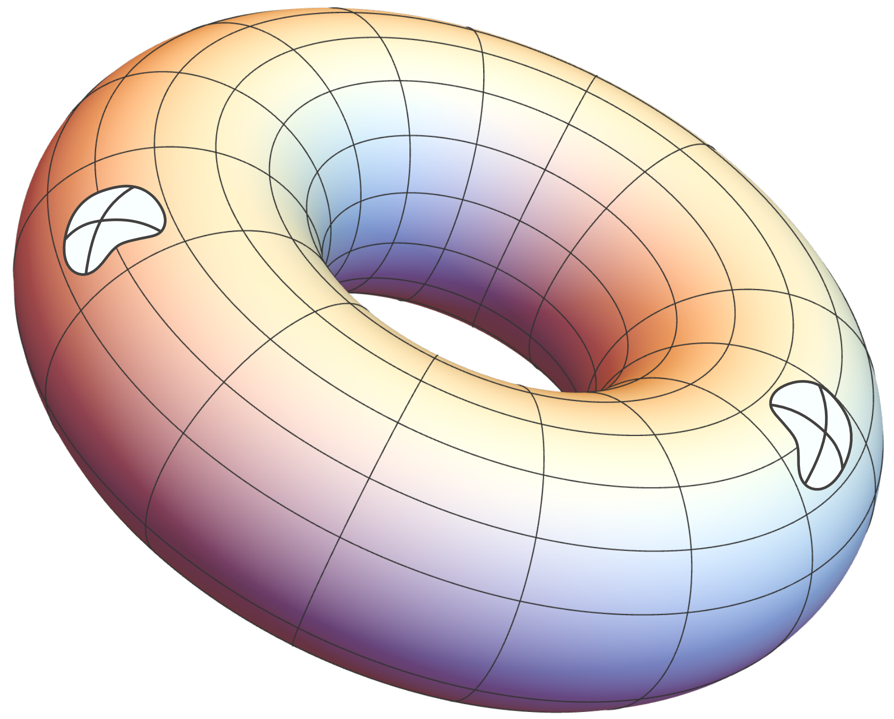
T_N block : $\mathcal{N} = 2$ SCFTs with $\mathrm{SU}(2) \times \mathrm{U}(1)_R \times \mathrm{SU}(N)^3$ global symmetry as world-volume theories of stack of M5 on three-punctured sphere.

General introduction

- Wrapping M5s on Riemann surfaces and T_N blocks

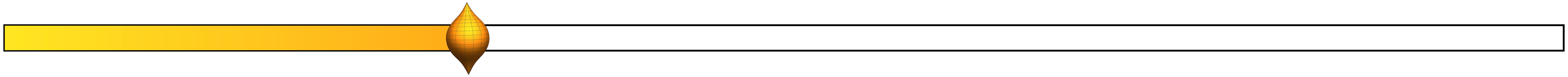
Gluing T_N blocks is gauging some $\mathrm{SU}(N)$: higher genus Riemann surfaces





$\Sigma_{1,2}$

S-class: gluing with $\mathcal{N} = 2$ vector multiplet



General introduction

- The B3W model

Up to now, compactification on Riemann surface. Generalization to wrapping branes on calibrated cycles on CYs. Calibration needed for twisting, aka preserve SUSY

General introduction

- The B3W model

Further generalization [Bah, Beem, Bobev, Wecht ’12]:

$$\begin{tikzcd} \mathbb{C}^2 \arrow[r, hook] & CY_3 \arrow[d, "\pi"] \\ & C_g \end{tikzcd}$$

$$CY_3 = \mathrm{Tot}\, V$$

$$\det V = K_{C_g}$$

To preserve SUSY

$$\begin{tikzcd} \mathrm{SU}(2) \arrow[r] & V \arrow[d, "\pi"] \\ & C_g \end{tikzcd}$$

General introduction

- The B3W model

Further generalization [Bah, Beem, Bobev, Wecht '12]:

IR dynamics of branes wrapped on this geometry depend on choice of this rank-2 vector bundle

$$
\begin{array}{ccc}
\mathrm{SU}(2) & \longrightarrow & V \\
 & & \Big\downarrow \pi \\
 & & C_g
\end{array}
$$

General introduction

- The B3W model

Reduce structure group from $\mathrm{SU}(2)$ to $\mathrm{U}(1)$

CY_3 decomposable $\mathscr{L}_1 \oplus K_{C_g}\mathscr{L}_2 \to C_g$

$c_1(\mathscr{L}_1) = p,\ c_1(\mathscr{L}_2) = q,\ p + q = 2g - 2$

Manifest $\mathrm{U}(1)^2$ isometry

$$\begin{tikzcd} \mathbb{C}^2 \arrow[r, hook] & \mathcal{L}_1 \oplus \mathcal{L}_2 \arrow[d, "\pi"] \\ & C_g \end{tikzcd}$$

General introduction

- The B3W model

Limiting cases $\quad c_1(\mathscr{L}_1) = p,\ c_1(\mathscr{L}_2) = q,\ p + q = 2g - 2$

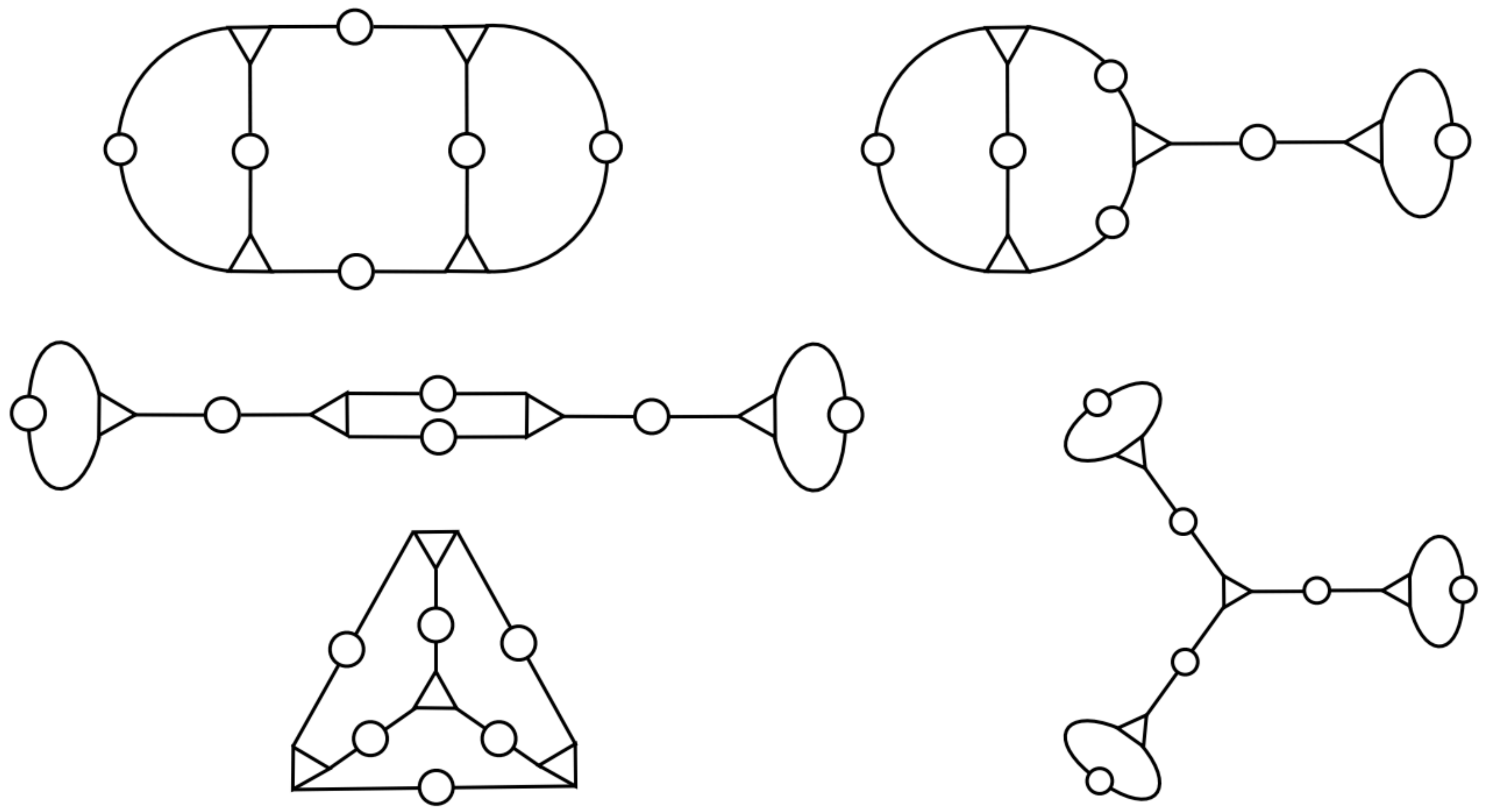
$q = 0$ or $p = 0 \implies X = \mathbb{C} \times T^{\star}C_g,\ \mathcal{N} = 2$ MN theories

$q = p,\ \mathcal{N} = 1$ Sicilian gauge theories [Benini, Tachiwaka, Wecht ’09]

General introduction

- The B3W model

General p, q can be constructed from opportune gluing of $2(g-1)$ T_N blocks to form a Riemann surface with no punctures. Gluing with both $\mathcal{N} = 1{,}2$ vector multiplets $\Longrightarrow$ choice of p, q



Outline

General introduction

Field Theory

- The anomaly polynomial of an M5-brane
- Stacking the branes
- Wrapping the branes
- Two-dimensional central charge

The gravity dual

Field Theory

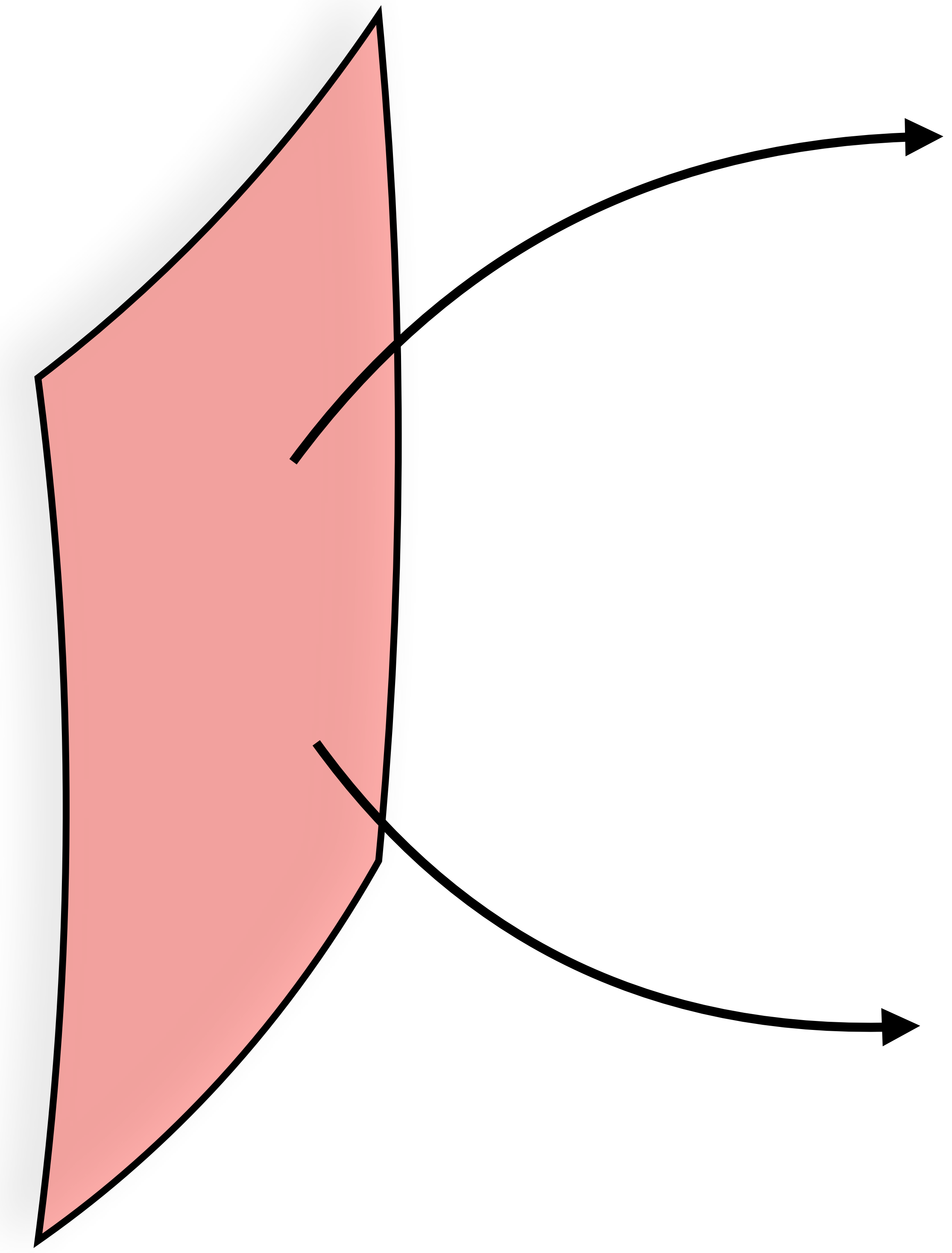
- The anomaly polynomial of an M5-brane

Supersymmetric $\mathcal{N} = (2{,}0)$ abelian tensor multiplet in $6d$:

- Two-form with self-dual field strength
- 5 scalars
- 4 real Weyl fermions

Field Theory

- The anomaly polynomial of an M5-brane



Four components chiral spinors

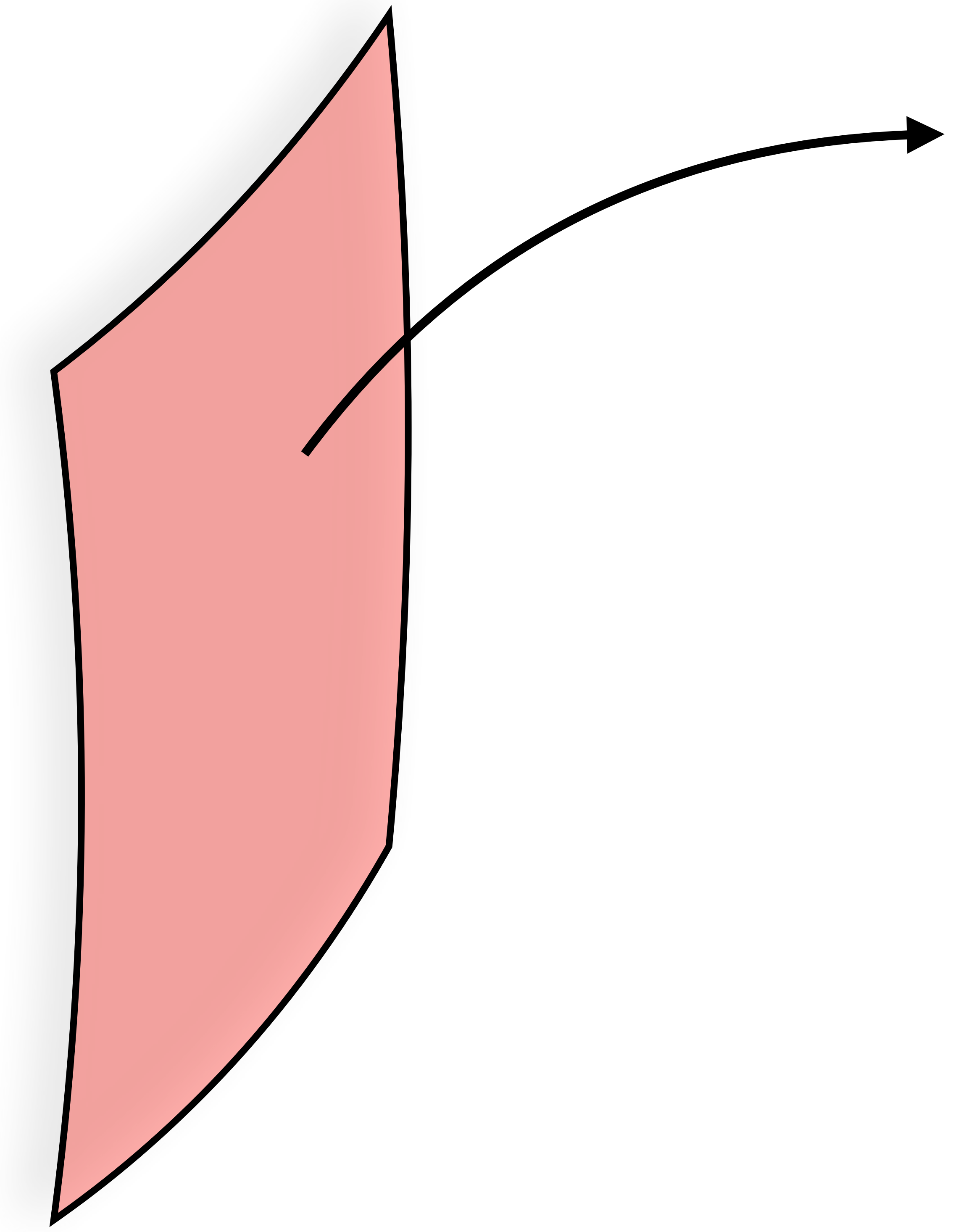
$$I_D = \frac{1}{2}\mathrm{ch}S(N)\,\hat{A}(TW)$$

Self-dual chiral two-form

$$I_A = \frac{1}{5760}\left(16p_1(TW)^2 - 112p_2(TW)\right)$$

Field Theory

- The anomaly polynomial of an M5-brane



Four components chiral spinors

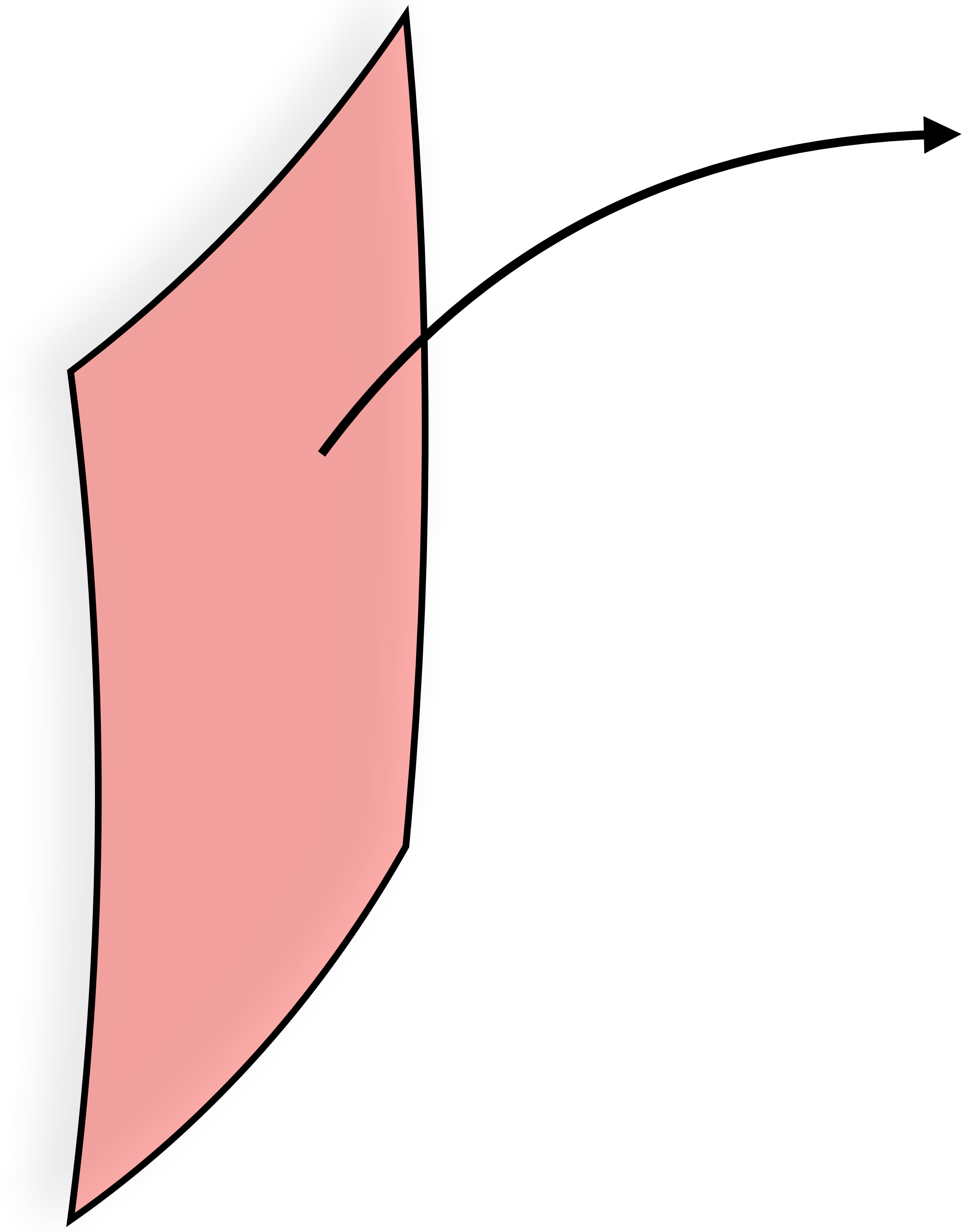
$$I_D = \frac{1}{2}\mathrm{ch}S(N)\,\hat{A}(TW)$$

Sections of rank-four spinor bundle constructed from the normal bundle N using the spinor rep of $SO(5)$

$SO(5)$ is the remaining isometry from M-theory after M5 defect insertion

Field Theory

- The anomaly polynomial of an M5-brane



Four components chiral spinors

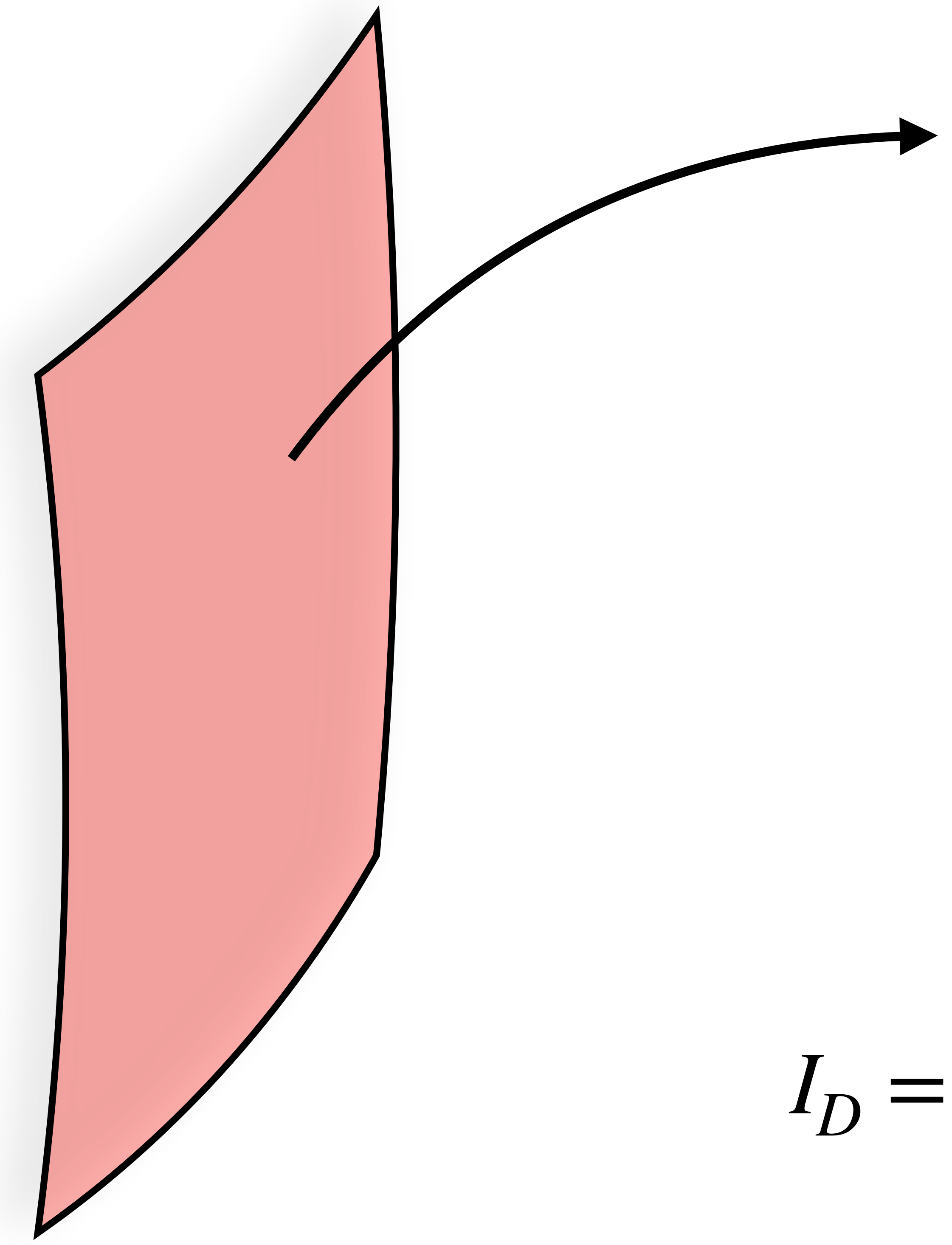
$$I_D = \frac{1}{2} \mathrm{ch} S(N) \, \hat{A}(TW)$$

$\hat{A}(TW)$ is the Dirac genus of TW, index of the Dirac operator on it

$$\hat{A}(TW) = 1 - \frac{p_1(TW)}{24} + \frac{7p_1(TW)^2 - 4p_2(TW)}{5760}$$

Field Theory

- The anomaly polynomial of an M5-brane



Four components chiral spinors

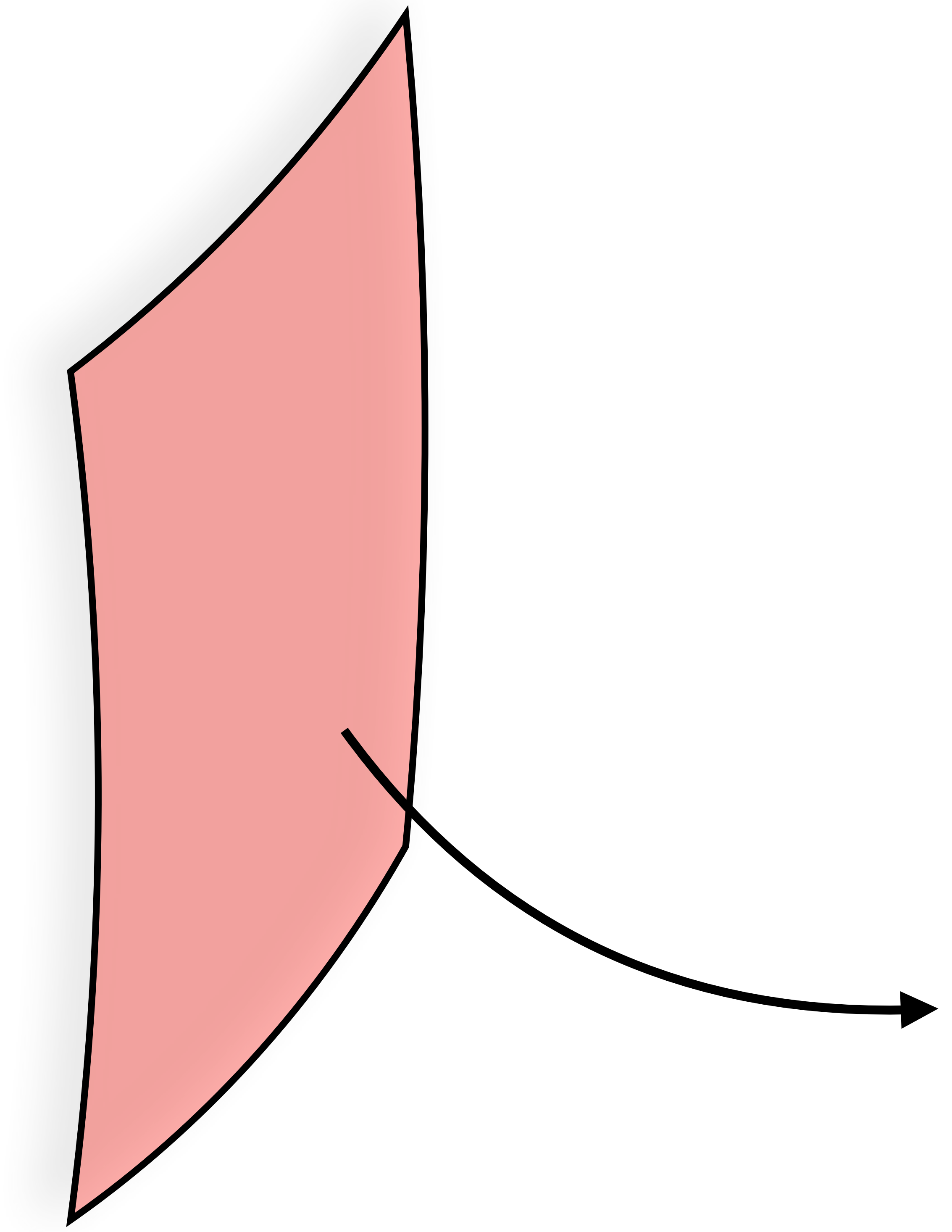
$$I_D = \frac{1}{2}\mathrm{ch}S(N)\,\hat{A}(TW)$$

$$\Downarrow$$

$$I_D = \frac{1}{2}\left(\frac{p_2(N)^2}{24} + \frac{p_1(N)^2}{96} - \frac{p_1(N)p_1(TW)}{48} + \frac{7p_1(TW)^2 - 4p_2(TW)}{1440}\right)$$

Field Theory

- The anomaly polynomial of an M5-brane



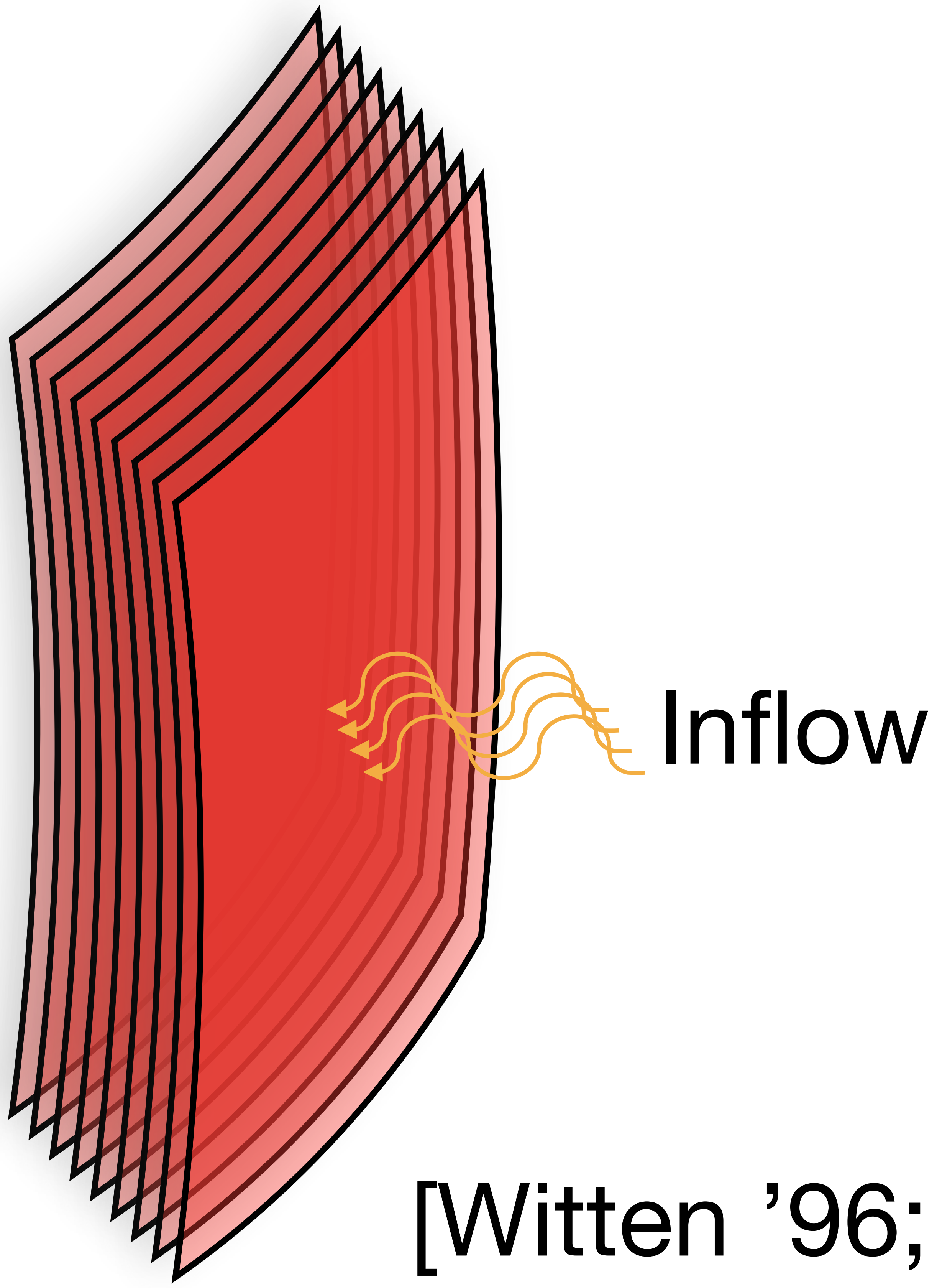
Propagates on brane world-volume W, does not see normal bundle

Self-dual chiral two-form

$$I_A = \frac{1}{5760}\left(16p_1(TW)^2 - 112p_2(TW)\right)$$

Field Theory

- Stacking the branes



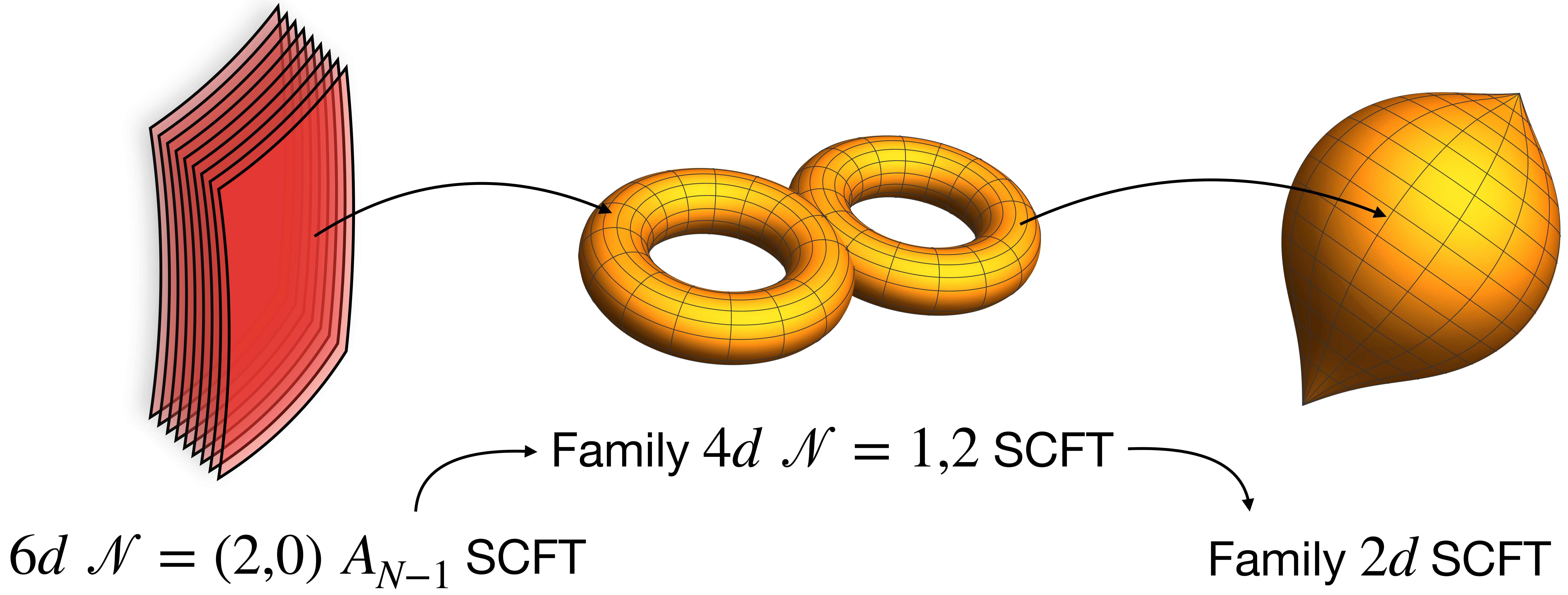
Anomaly polynomial for A_N case

$$I_8 = \underbrace{\frac{N-1}{48}\left[p_2(NW) - p_2(TW) + \frac{1}{4}(p_1(TW) - p_1(NW))^2\right]}_{\text{Spinors+Three-form}} + \underbrace{\frac{N^3-N}{24}p_2(NW)}_{\text{CS term}}$$

[Witten ’96; Harvey, Minasian, Moore ’98; Intriligator ’00; Yi ’01; …]

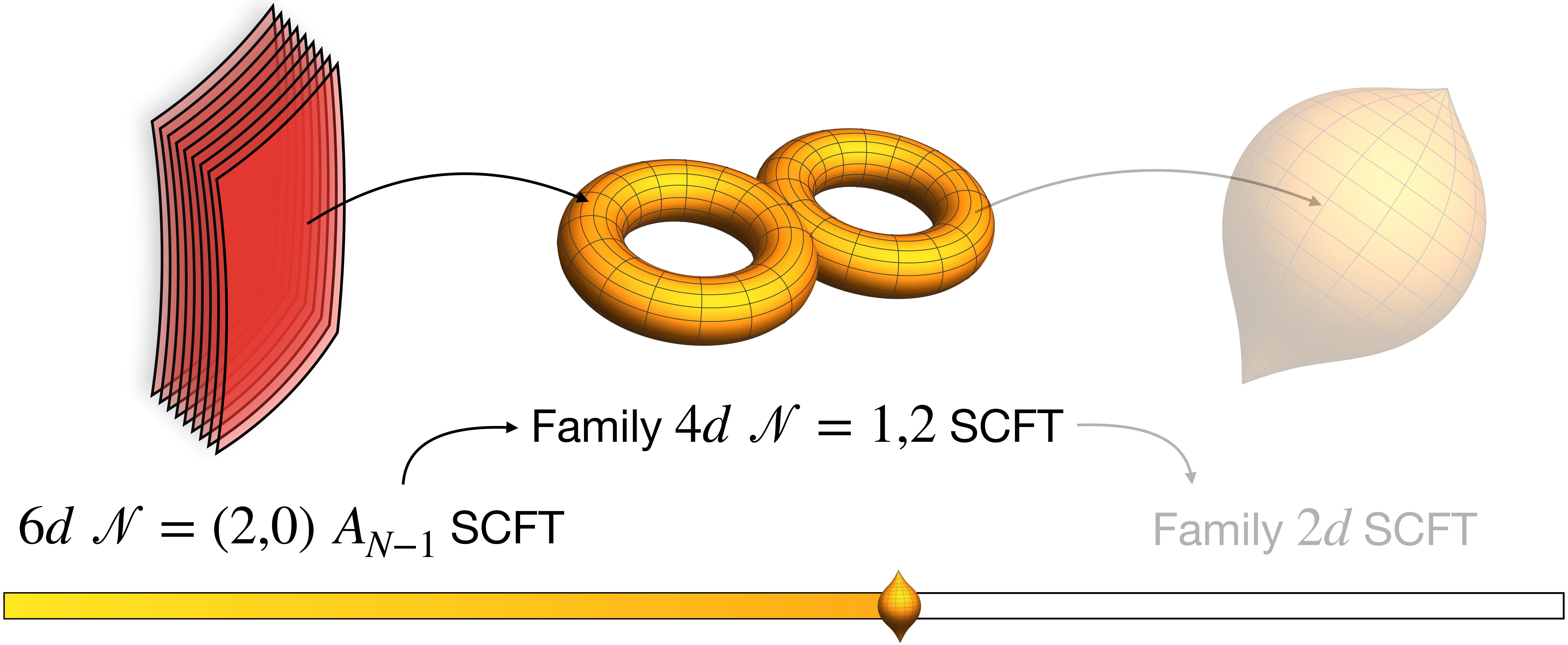
Field Theory

- Wrapping the branes



Field Theory

- Wrapping the branes



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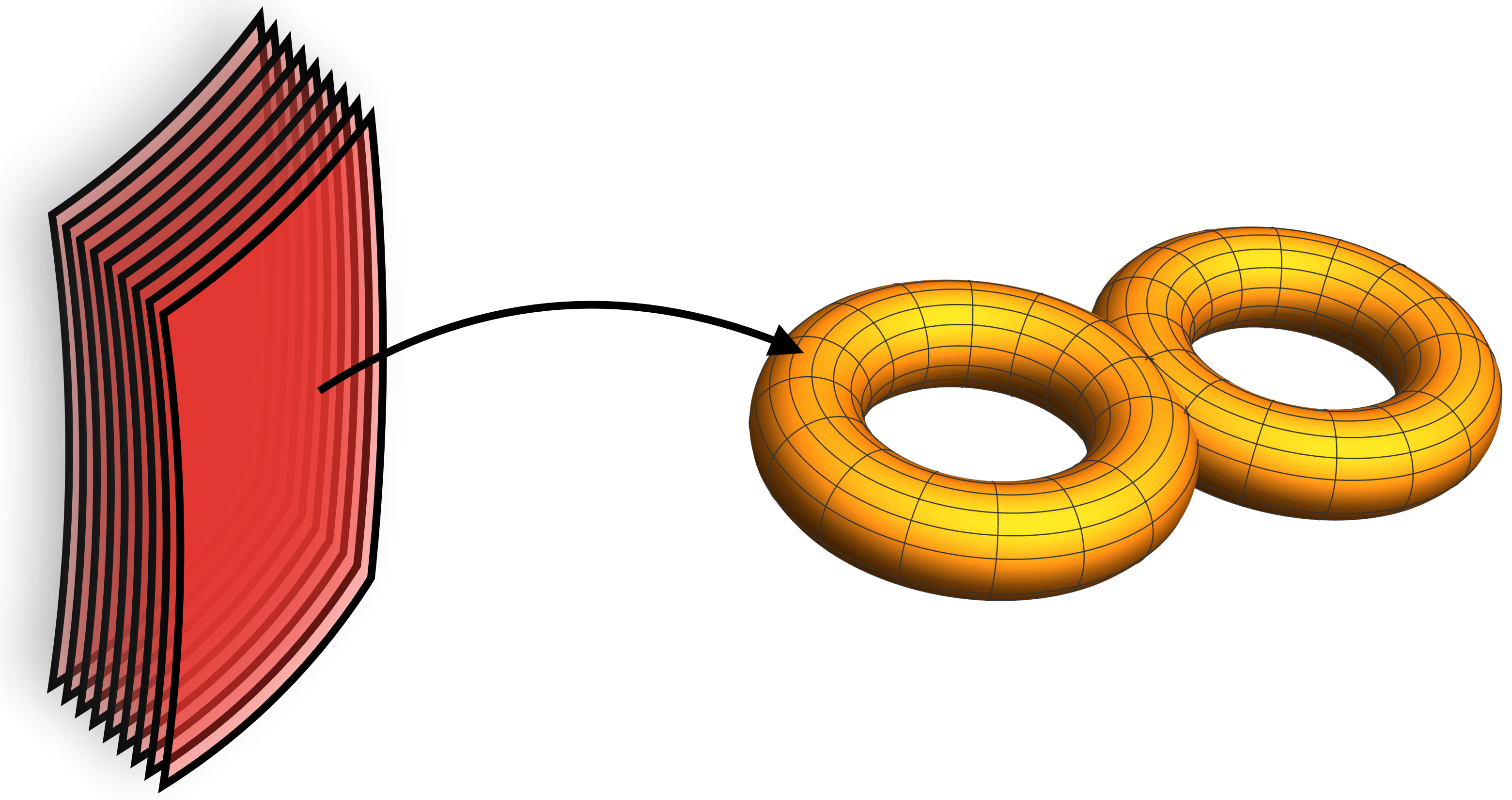


[Bah, Beem, Bobev, Wecht '12]

$$I_6 = \int_{C_g} I_8 \sim \sum_{i,j,k=1,2} A_{ijk} c_1(F_i) c_1(F_j) c_1(F_k)$$

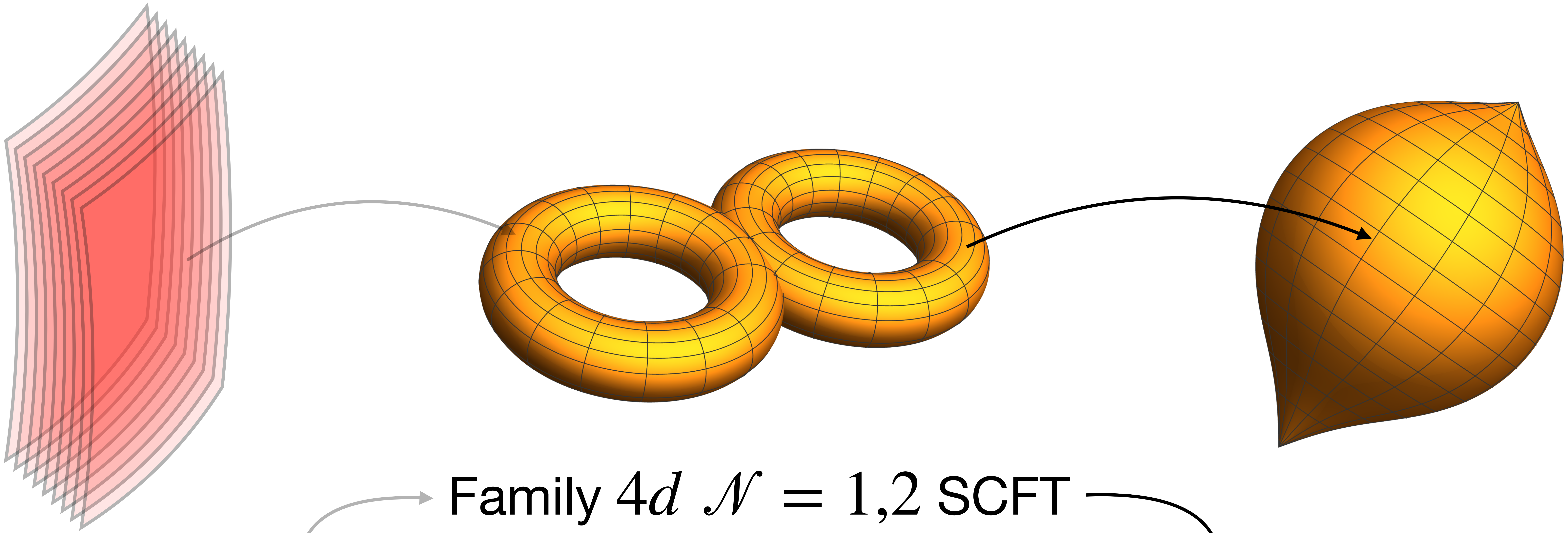
Where A_{ijk} abelian anomalies of $4d$ theory

$$A_{RRR} = (g-1)N^3 \qquad A_{RRF} = -\frac{1}{3}(g-1)zN^3$$

$$A_{RFF} = -\frac{1}{3}(g-1)N^3 \qquad A_{FFF} = (g-1)zN^3$$

Field Theory

- Wrapping the branes



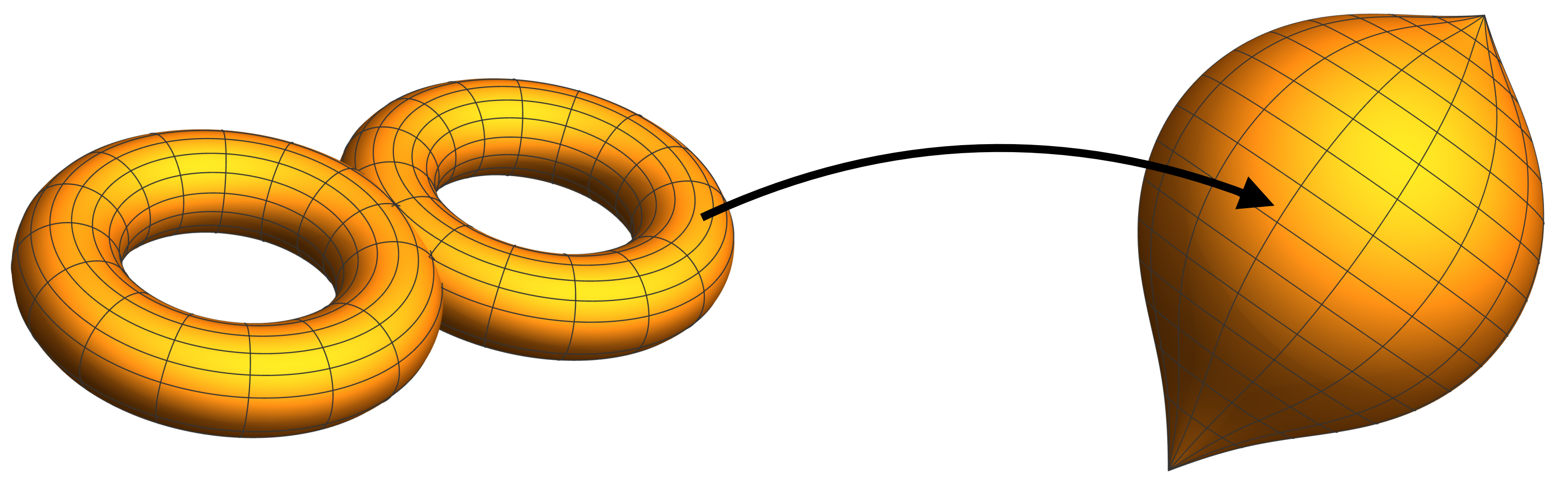
Family $4d$ $\mathcal{N} = 1{,}2$ SCFT

$6d$ $\mathcal{N} = (2{,}0)$ A_{N-1} SCFT

Family $2d$ SCFT

Field Theory

- Two-dimensional central charge



[Amariti, Mancani, DM, Petri, Segati ’23]

Additional abelian symmetry from azimuthal rotation on spindle

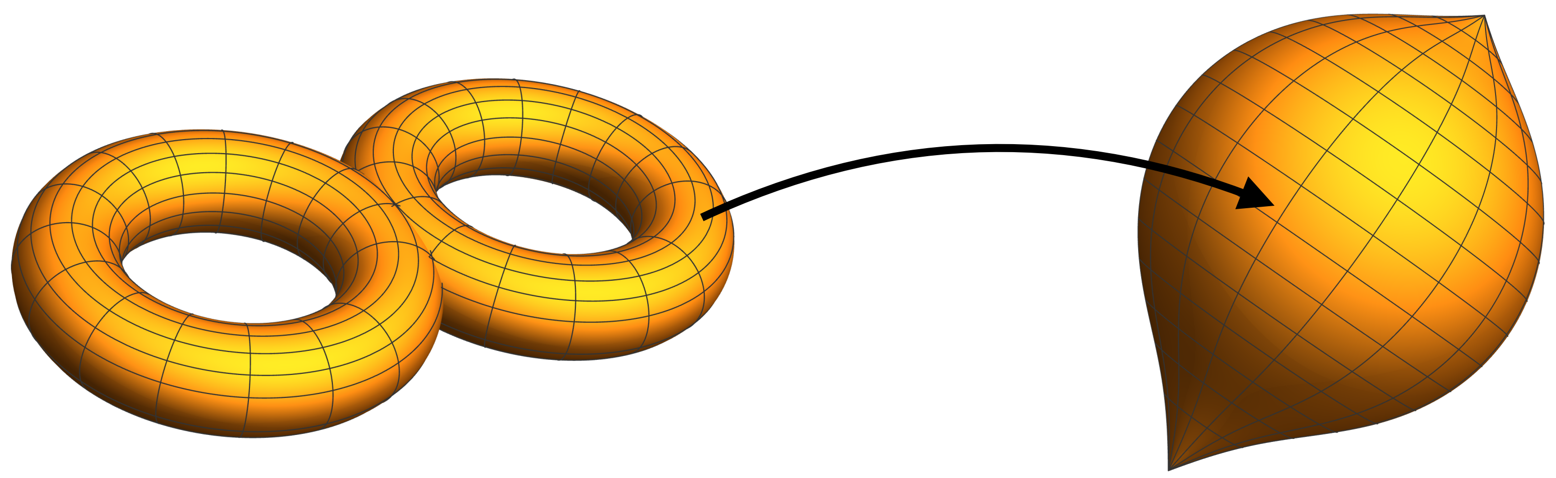
$$I_4 = \int_{\mathbb{WCP}^1_{[n_N,n_S]}} I_6 =$$

Subleading at large N

$$\sim \frac{c_R}{6} c_1(F_R)^2 - \cancel{\frac{c_R - c_L}{24}} p_1(TW_2)$$

Field Theory

- Two-dimensional central charge



[Amariti, Mancani, DM, Petri, Segati ’23]

Fix magnetic fluxes

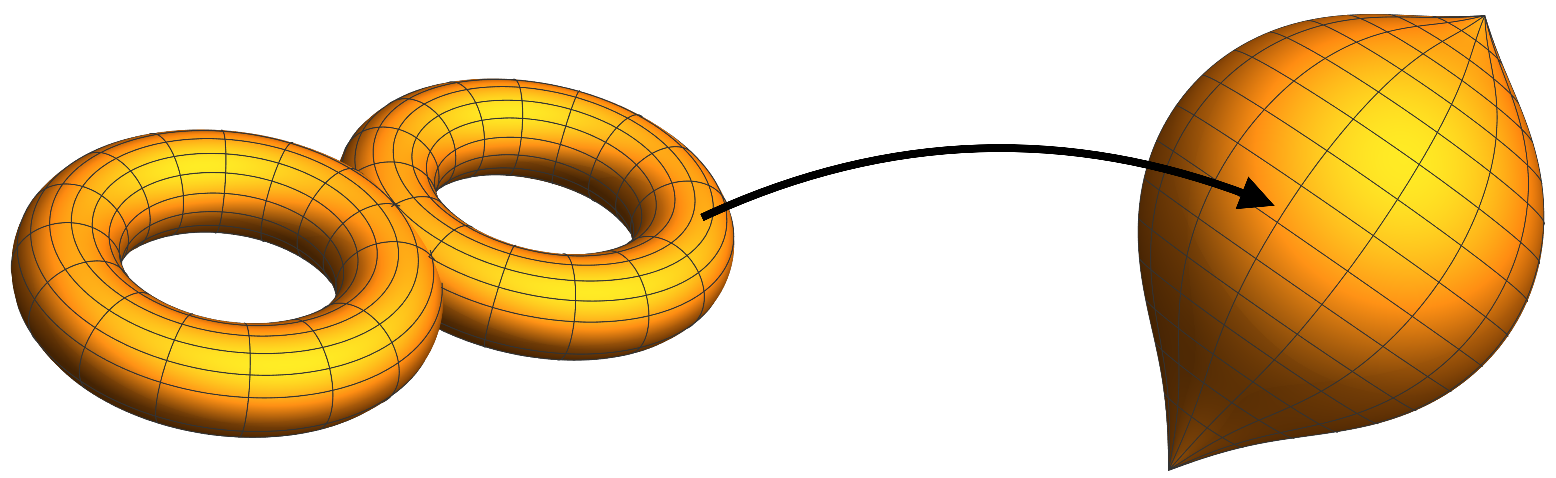
$$\int c_1(F_R) = [\rho_R]_N^S = \frac{p_R}{n_N n_S}$$

$$\int c_1(F_F) = [\rho_F]_N^S = \frac{p_F}{n_N n_S}$$

Preserve SUSY by R-symmetry (anti-)twist

Field Theory

- Two-dimensional central charge



[Amariti, Mancani, DM, Petri, Segati ’23]

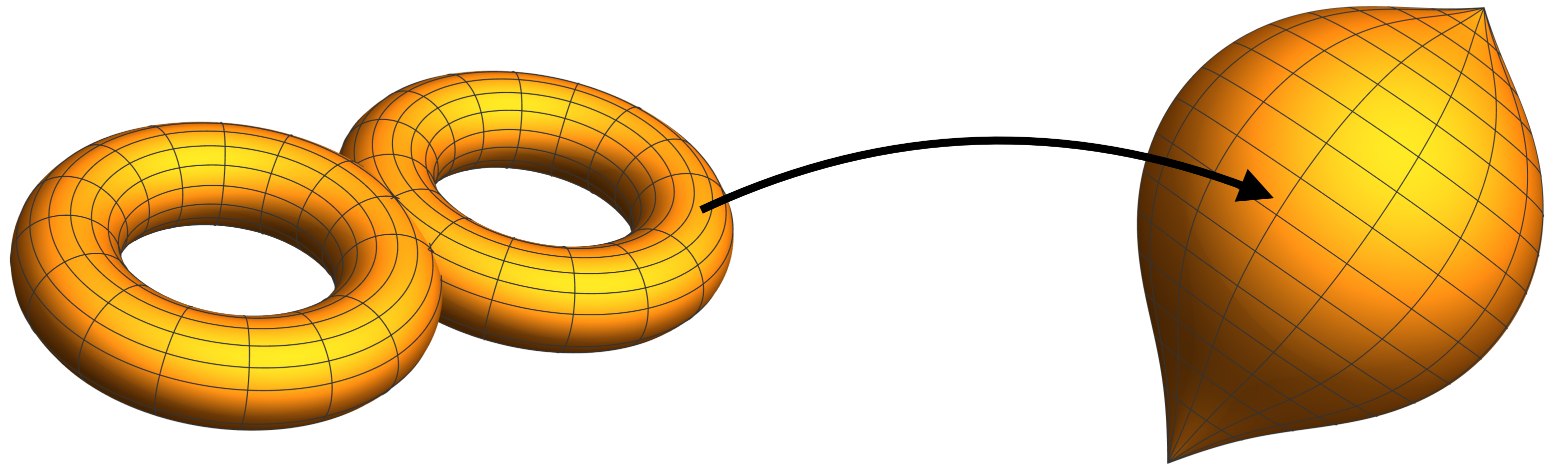
$$\rho_R(y_N) = \frac{(-1)^{t_N}}{n_N} \qquad \rho_R(y_S) = \frac{(-1)^{t_S+1}}{n_S}$$

Where $t_N = 0,1$. Twist $t_S = t_N$, anti-twist $t_S = t_N + 1$

Flavour flux fixed up to arbitrary constant

Field Theory

- Two-dimensional central charge



[Amariti, Mancani, DM, Petri, Segati ’23]

Central charge in large-N from anomaly polynomial and allow mixing

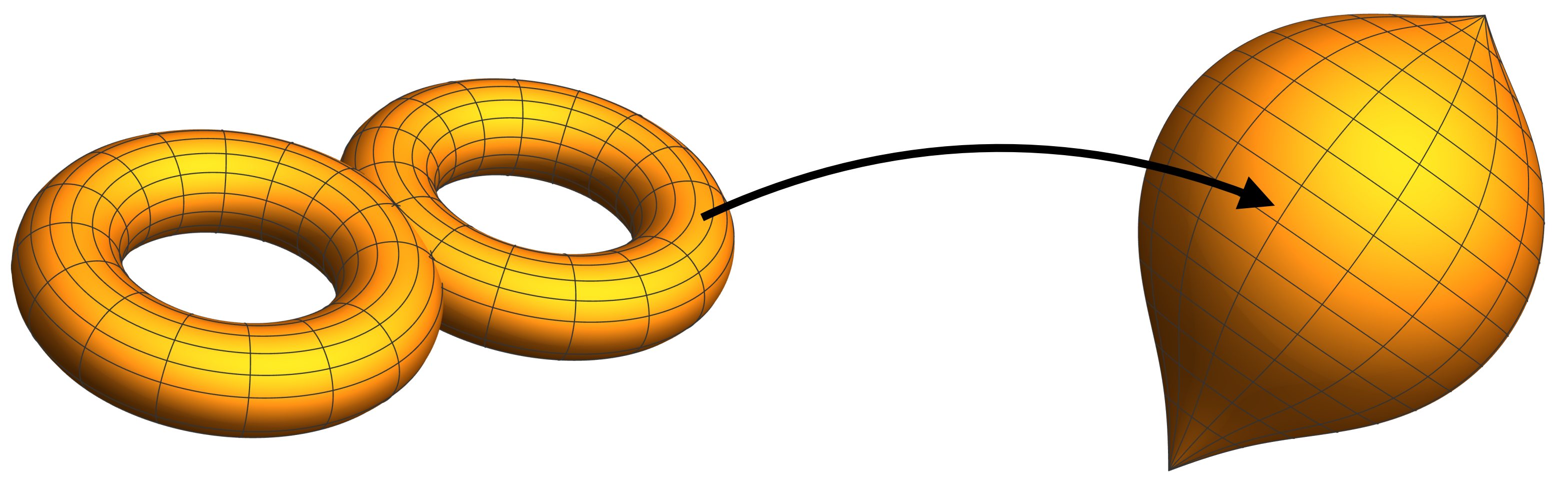
$$R^{\mathrm{trial}}(x,\epsilon) = R + xF + \epsilon J$$

$$c_R(\epsilon, x) = \frac{6 I_4(\epsilon, x)}{c_1(F_R)^2}$$

C-extremization!

Field Theory

- Two-dimensional central charge



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C-extremization!

Field Theory

- Two-dimensional central charge

N.B. $z = \dfrac{p-q}{p+q}$

Twist

$$\frac{(g-1)N^3\left(4p_F^2-\left(n_N+n_S\right)^2\right)\left(2zp_F+(-1)^{t_N}\left(n_N+n_S\right)\right)\left((-1)^{t_N}\left(n_N+n_S\right)\left(16zp_F+\left(z^2+3\right)(-1)^{t_N}\left(n_N+n_S\right)\right)+4\left(3z^2+1\right)p_F^2\right)}{2n_Nn_S\left(8p_F^2\left(-2n_Nn_S+3z^2n_S^2+3z^2n_N^2\right)-32zp_F^3(-1)^{t_N}\left(n_N+n_S\right)+8zp_F(-1)^{t_N}\left(n_N+n_S\right)\left(3n_N^2-2n_Nn_S+3n_S^2\right)-48z^2p_F^4+\left(n_N+n_S\right)^2\left(-2\left(z^2+2\right)n_Nn_S+\left(z^2+4\right)n_S^2+\left(z^2+4\right)n_N^2\right)\right)}$$

Anti-twist

$$\frac{(g-1)N^3\left(\left(n_S-n_N\right)^2-4p_F^2\right)\left(2zp_F+(-1)^{t_N}\left(n_N-n_S\right)\right)\left((-1)^{t_N}\left(n_N-n_S\right)\left(16zp_F+\left(z^2+3\right)(-1)^{t_N}\left(n_N-n_S\right)\right)+4\left(3z^2+1\right)p_F^2\right)}{2n_Nn_S\left(8p_F^2\left(2n_Nn_S+3z^2n_S^2+3z^2n_N^2\right)+32zp_F^3(-1)^{t_N}\left(n_S-n_N\right)-8zp_F(-1)^{t_N}\left(n_S-n_N\right)\left(3n_N^2+2n_Nn_S+3n_S^2\right)-48z^2p_F^4+\left(n_S-n_N\right)^2\left(2\left(z^2+2\right)n_Nn_S+\left(z^2+4\right)n_S^2+\left(z^2+4\right)n_N^2\right)\right)}$$

Field Theory

- Two-dimensional central charge

N.B. $z = \dfrac{p-q}{p+q}$

Twist

$$
\frac{(g-1)N^3\left(4p_F^2-\left(n_N+n_S\right)^2\right)\left(2zp_F+(-1)^{t_N}\left(n_N+n_S\right)\right)\left((-1)^{t_N}\left(n_N+n_S\right)\left(16zp_F+\left(z^2+3\right)(-1)^{t_N}\left(n_N+n_S\right)\right)+4\left(3z^2+1\right)p_F^2\right)}{2n_Nn_S\left(8p_F^2\left(-2n_Nn_S+3z^2n_S^2+3z^2n_N^2\right)-32zp_F^3(-1)^{t_N}\left(n_N+n_S\right)+8zp_F(-1)^{t_N}\left(n_N+n_S\right)\left(3n_N^2-2n_Nn_S+3n_S^2\right)-48z^2p_F^4+\left(n_N+n_S\right)^2\left(-2\left(z^2+2\right)n_Nn_S+\left(z^2+4\right)n_S^2+\left(z^2+4\right)n_N^2\right)\right)}
$$

Anti-twist

$$
\frac{(g-1)N^3\left(\left(n_S-n_N\right)^2-4p_F^2\right)\left(2zp_F+(-1)^{t_N}\left(n_N-n_S\right)\right)\left((-1)^{t_N}\left(n_N-n_S\right)\left(16zp_F+\left(z^2+3\right)(-1)^{t_N}\left(n_N-n_S\right)\right)+4\left(3z^2+1\right)p_F^2\right)}{2n_Nn_S\left(8p_F^2\left(2n_Nn_S+3z^2n_S^2+3z^2n_N^2\right)+32zp_F^3(-1)^{t_N}\left(n_S-n_N\right)-8zp_F(-1)^{t_N}\left(n_S-n_N\right)\left(3n_N^2+2n_Nn_S+3n_S^2\right)-48z^2p_F^4+\left(n_S-n_N\right)^2\left(2\left(z^2+2\right)n_Nn_S+\left(z^2+4\right)n_S^2+\left(z^2+4\right)n_N^2\right)\right)}
$$

Outline

General introduction

The boundary

The bulk

- Consistent AdS_5 truncation with hypermultiplets
- Down to $AdS_3 \times \mathbb{WCP}^1_{[n_N,n_S]}$
- Central charge from the poles and matching with field theory
- Numerical solutions

The gravity side

- Consistent AdS_5 truncation with hypermultiplets

Starting point: consistent $5d$ truncation from $D = 11$ of [Cassani, Josse, Petrini, Waldram '21]

- One hypermultiplet
- Two vector multiples

Gauge group $\mathrm{U}(1) \times \mathbb{R}$

The gravity side

- Consistent AdS_5 truncation with hypermultiplets

Ansatz $\mathrm{d}s^2_{11} = e^{2\Delta}\mathrm{d}s^2_{\mathrm{AdS}_5} + \mathrm{d}s^2_6$ warped product $\mathrm{AdS}_5 \times_w M_6$ where second factor is squashed four-sphere fibered over Riemann surface C_g

$$\begin{array}{ccc} M_4 & \hookrightarrow & M_6 \\ & & \downarrow \pi \\ & & C_g \end{array}$$

Dependence of metric on factors p, q introduced before & scalar curvature k of C_g

Generalizes $\mathcal{N} = 1{,}2$ twistings of MN [Maldacena, Nunez '00]

The gravity side

- Consistent AdS_5 truncation with hypermultiplets

Vector multiplet: two real scalars Σ, ϕ parametrize $M_V = \mathbb{R}_+ \times \mathrm{SO}(1,1)$

Hypermultiplet: four scalars $\varphi, \Xi, \theta_1, \theta_2$ parametrize $M_H = \dfrac{\mathrm{SU}(2,1)}{\mathrm{SU}(2) \times \mathrm{U}(1)}$

Further truncation: $\theta_1 = \theta_2 = 0$

Introduce superpotential $W = \dfrac{\Sigma^3((\mathbf{k}e^{2\varphi} + 4)\cosh\phi - \mathbf{zk}e^{2\varphi}\sinh\phi) + e^{2\varphi}}{4\Sigma^2}$

The gravity side

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The gravity side

- Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N,n_S]}$

Ansatz $\mathrm{d}s^2 = e^{2V(y)}\mathrm{d}s^2_{\mathrm{AdS}_3} + f(y)^2\mathrm{d}y^2 + h(y)^2\mathrm{d}z^2$ where (y, z) are coordinates on Spindle: $z \sim z + 2\pi$ and $y \in [y_N, y_S]$

Gauge fields: $A^{(I)} = a^{(I)}(y)\mathrm{d}z$ where $I = 1,2$

Assume: $\Sigma(y), \phi(y), \varphi(y)$ and $\Xi = \bar{\Xi} \cdot z$

The gravity side

- Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N,n_S]}$

Orthonormal frame of reference [Arav, Gauntlett, Roberts, Rosen ’22]

$$e^a = e^V \bar{e}^a, \qquad e^3 = f\,\mathrm{d}y, \qquad e^4 = h\,\mathrm{d}z$$

Field strength becomes

$$f\,h\,F^{(I)}_{34} = \partial_y a^{(I)}$$

The gravity side

- Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N,n_S]}$

Constants

Then, Maxwell's equations are

$$\frac{2e^{3V}}{3\Sigma^2}\left[(\cosh 2\phi - \mathbf{z}\sinh 2\phi)F^{(1)}_{34} + (\mathbf{z}\cosh 2\phi - \sinh 2\phi)F^{(2)}_{34}\right] = \mathscr{E}_1$$

$$\frac{2e^{3V}}{3\Sigma^2}\left[\mathbf{zk}\Sigma^6 F^{(0)}_{34} - (\cosh 2\phi + \mathbf{z}\sinh 2\phi)F^{(1)}_{34} + (\mathbf{z}\cosh 2\phi + \sinh 2\phi)F^{(2)}_{34}\right] = \mathscr{E}_2$$

$$\partial_y\left(\frac{1}{3}e^{3V}\Sigma^4 F^{(0)}_{34}\right) = \frac{1}{4}e^{4\psi+3V} g\, f\, h^{-1} D_z \Xi$$

The gravity side

- Down to $AdS_3 \times \mathbb{WCP}^1_{[n_N, n_S]}$

Killing spinor $\epsilon = \psi \otimes \chi$ with ψ on AdS_3 and χ on Spindle, where $\nabla_m \psi = -\frac{\kappa}{2}\Gamma_m \psi$ and

$$\chi = e^{\frac{V}{2}} e^{isz} \begin{pmatrix} \sin\frac{\xi}{2} \\ \cos\frac{\xi}{2} \end{pmatrix}$$

The gravity side

- Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N,n_S]}$

BPS equations

$$\xi' - 2f(gW\cos\xi + \kappa e^{-V}) = 0$$

$$V' - \frac{2}{3} fgW\sin\xi = 0$$

$$\Sigma' + \frac{2}{3} fg\,\Sigma^2 \sin\xi\,\partial_\Sigma W = 0$$

$$\phi' + 2fg\sin\xi\,\partial_\phi W = 0$$

$$\varphi' + \frac{fg}{\sin\xi}\partial_\varphi W = 0$$

$$h' - \frac{2fh}{3\sin\xi}(gW(1+2\cos^2\xi) + 3\kappa e^{-V}\cot\xi) = 0,$$

The gravity side

- Central charge from the poles and matching with field theory

Conditions at poles are enough [Arav, Gauntlett, Roberts, Rosen ’22; Suh ’23; Amariti, Petri, Segati ‘23]

- $\varphi(y)$ finite at poles: $\partial_\varphi W|_{N,S} = 0 \implies \mathbf{k}\Sigma^3|_{N,S} + \dfrac{1}{\cosh\phi|_{N,S} - \mathbf{z}\sinh\phi|_{N,S}} = 0$

Combine two conserved charges as

$$Q_1|_{N,S} = \mathscr{E}_1|_{N,S} = \frac{4}{3}e^{2V|_{N,S}}\left(\frac{\kappa(\sinh(\phi|_{N,S}) - \mathbf{z}\cosh(\phi|_{N,S}))}{\Sigma|_{N,S}} - \mathbf{z}ge^{V|_{N,S}}\cos(\xi|_{N,S})\right)$$

$$Q_2|_{N,S} = \mathscr{E}_1|_{N,S} - \mathscr{E}_2|_{N,S} = \frac{4\kappa e^{2V|_{N,S}}}{3\Sigma|_{N,S}}\left(2\sinh(\phi|_{N,S}) - \mathbf{zk}\Sigma|^3_{N,S}\right).$$

The gravity side

- Central charge from the poles and matching with field theory

Fluxes

$$\frac{p_I}{n_N n_S} = \frac{1}{2\pi}\int_{\mathbb{WCP}} g F^{(I)} = g\mathcal{I}^{(I)}\big|_N^S \qquad \mathcal{I}^{(I)} \equiv -k e^V \cos\xi h^I$$

Flavour flux $\quad p_F = g n_N n_S \mathcal{I}^{(1)}\big|_N^S$

R-symmetry flux $\quad -g n_N n_S \mathcal{I}^{(2)}\big|_N^S = \frac{1}{2}(n_S(-1)^{t_N} + n_N(-1)^{t_S})$

Constraint $\quad \mathcal{I}^{(0)} + \mathbf{zk}\mathcal{I}^{(1)} - \mathbf{k}\mathcal{I}^{(2)} = 0$

The gravity side

- Central charge from the poles and matching with field theory

Three equations before fix boundary conditions for V, h, ϕ, Σ

$$e^{V(y)}f(y)h(y) = -\frac{k}{2\kappa}\left(e^{3V(y)}\cos\xi(y)\right)'$$

Very important!!

Central charge

$$c_{2d} = \frac{3R_{\mathrm{AdS}_3}}{2G_3} = \frac{3}{2G_5}\Delta z \int_{y_n}^{y_s} e^{V(y)}\,|\,f(y)h(y)\,|\,\mathrm{d}y$$

The gravity side

- Central charge from the poles and matching with field theory

Central charges match with the FT ones! Both for twist and anti-twist

We found analytic solution by restricting to graviton sector only for anti-twist case with $\mathbf{k} = -1$ and generic z matching [Ferrero et al. '21; Ferrero, Gauntlett, Sparks '22]. Graviton sector fixes p_F

The gravity side

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The gravity side

- Numerical solutions

For generic p_F (consistent with quantization) we find numerical solution by integrating BPS eqns. [Arav, Gauntlett, Roberts, Rosen ’22; Suh ’23; Amariti, Petri, Segati ‘23]

Still only solutions for $\mathbf{k} = -1$ and anti-twist

n_S	n_N	p_F	$\mathbf{z}$	φ_S	φ_N	Δy
1	3	0	2	-0.285076	-0.274493	1.83241
1	7	-1	2	-0.172372	-0.170589	2.39707
1	3	0	3	-0.555814	-0.542721	1.82303
1	5	-1	3	-0.300428	-0.300346	2.16012
1	9	3	$\frac{1}{3}$	0.463989	0.363277	2.57446
1	5	0	$\frac{1}{3}$	0.126802	0.124497	2.16392
1	7	2	$\frac{1}{2}$	0.484886	0.347516	2.3322
3	7	0	$\frac{1}{2}$	0.104192	0.103447	1.74866

Thank you

Backup slides

Field Theory

- Couple of details on $2d$ anomaly polynomial

Gravitational anomalies

Anomaly polynomial in $2d$ is $I_4 = \frac{c_R}{6} c_1(F_R)^2 - \frac{c_R - c_L}{24} p_1(TW_2)$

In terms of $2d$ mixed anomalies $I_4 = \frac{1}{2} A_{ij} c_1(F_i) c_1(F_j) - \frac{k}{24} p_1(TW_2)$

Central charge of trial R-symmetry, many $\mathrm{U}(1)$s

$$c_R^{\mathrm{trial}}(t) = 3\left(A_{RR} + 2\sum_{i \neq R} t_i A_{iR} + \sum_{i,j \neq R} t_i t_j A_{ij} \right)$$

Field Theory

- Couple of details on $2d$ anomaly polynomial

In our case only $\mathrm{U}(1)_F$ and $\mathrm{U}(1)_J$, so mixing is

$$c_R(x,\epsilon) = 3(A_{RR} + 2\epsilon A_{RJ} + 2xA_{RF} + x^2\epsilon A_{FF} + \epsilon^2 A_{JJ} + 2x\epsilon A_{FJ})$$

where the anomalies are suitably normalized

The gravity side

- Numerical solutions