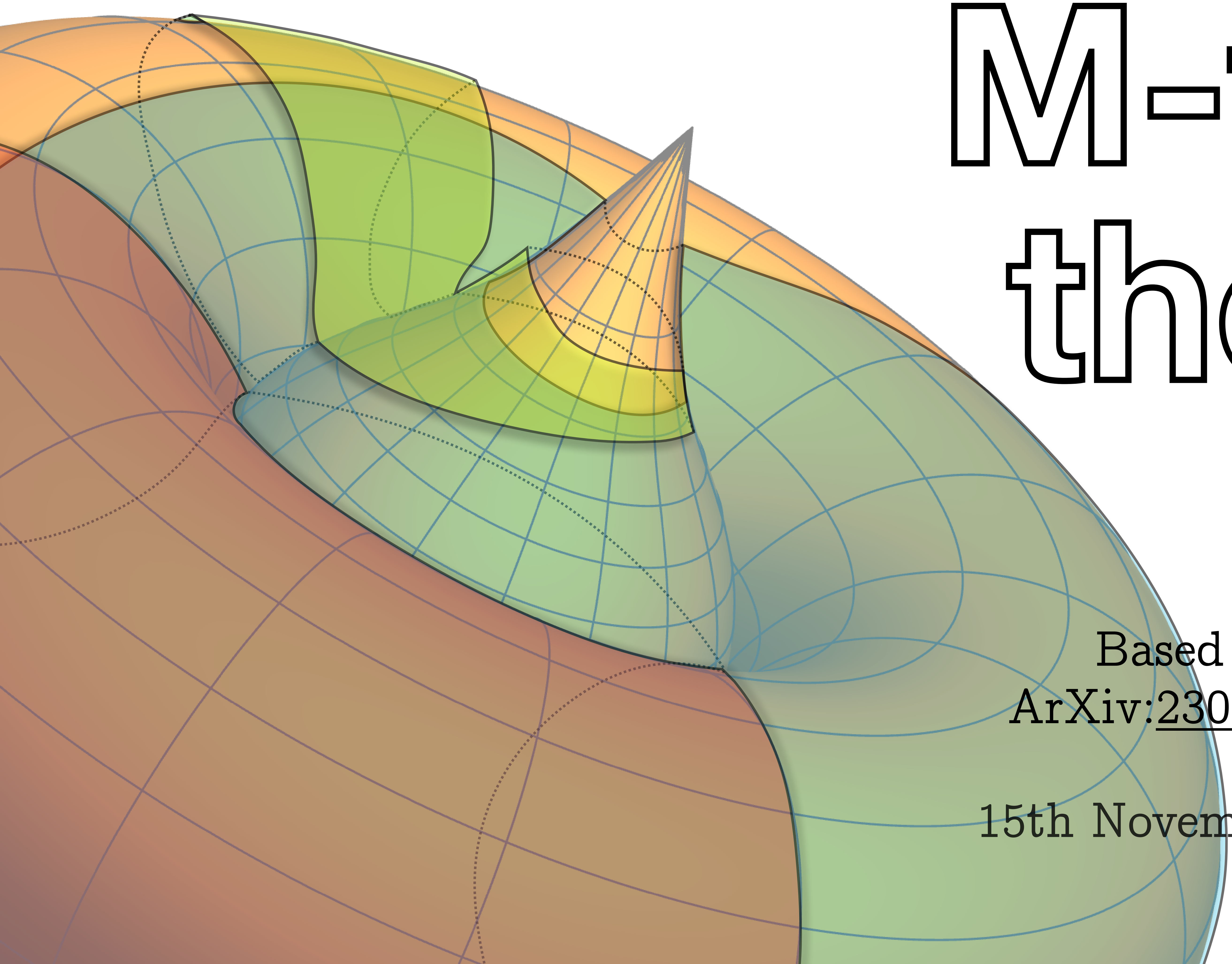
Università degli Studi di Milano

M-theory on the Spindle



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15th Novembre 2024

Outline

General introduction

The bulk

The boundary

Outline

General introduction

- Spindle geometry $\mathbb{WCP}^1_{[n_N,n_S]}$: Twist and anti-twist
- The M5 world-volume theory: $6d\ \mathcal{N} = (2,0)\ A_{N-1}$ SCFT
- Wrapping M5s on Riemann surfaces and T_N blocks
- The B3W model

The bulk

The boundary

Spindle Geometry

Important insights into strongly coupled SCFT by realizing them as RG fixed points of **compactification** of higher-dimensional QFTs

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Foundational work Maldacena & Nunez $4d$ SCFT from M5-branes on Riemann surface $\Sigma_g \implies$ SUSY preserved by **topological twist**

[Maldacena, Nunez (2000)]

Spindle Geometry

Important insights into strongly coupled SCFT by realizing them as RG fixed points of **compactification** of higher-dimensional QFTs

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[Maldacena, Nunez (2000)]

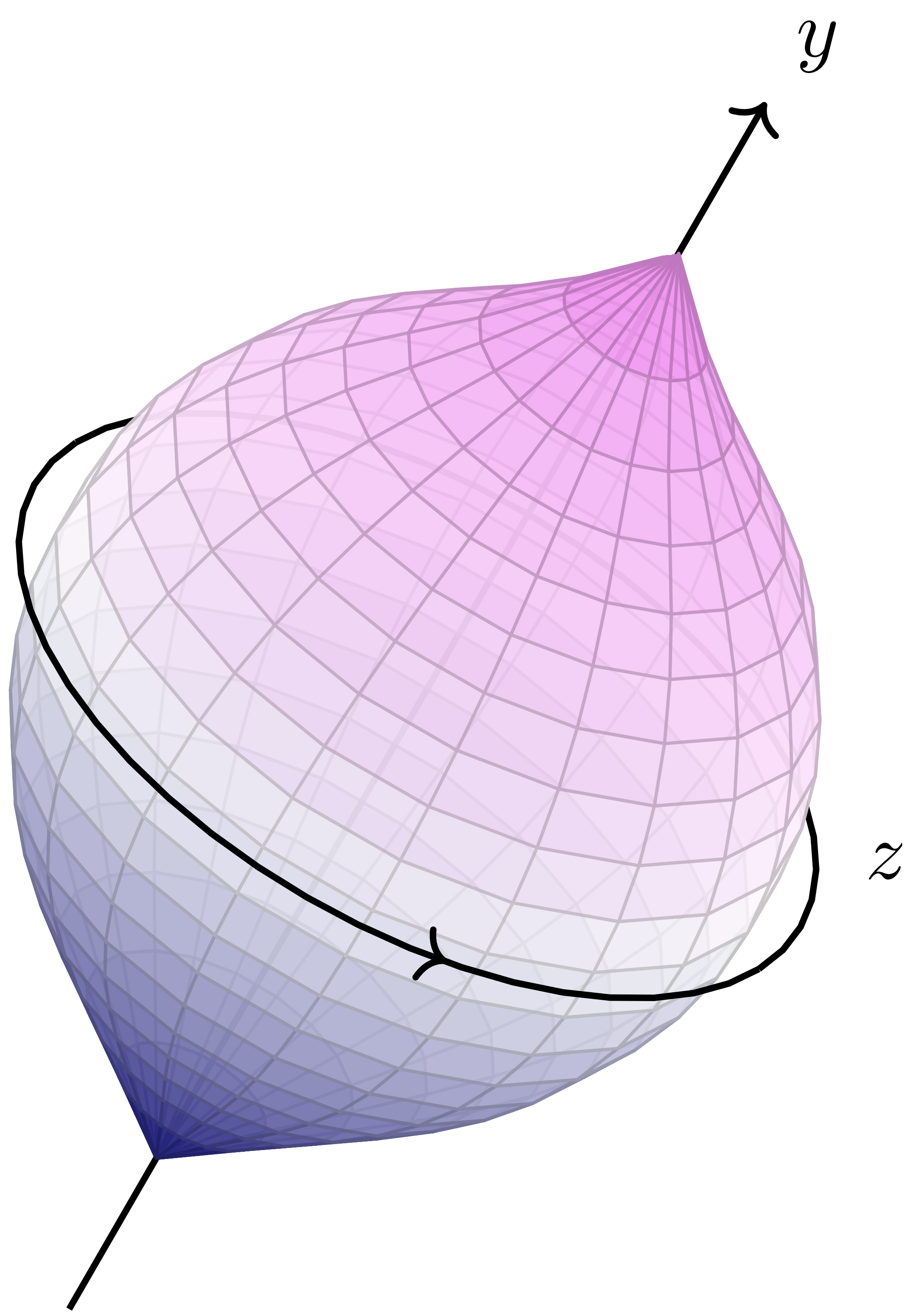
No covariantly constant spinor $(\partial_\mu + \omega_\mu)\epsilon = 0$, couple to background R-symmetry $A_\mu^R = -\omega_\mu$, then $(\partial_\mu + \cancel{\omega_\mu} + \cancel{A_\mu^R})\epsilon = 0 \implies \epsilon$ constant

Spindle Geometry

Condition $A_\mu = -\omega_\mu$ equivalent to choosing right flux for R-symmetry background

$$\frac{1}{2\pi}\int_{\Sigma_g} F^R = \chi(\Sigma_g) = 2(g-1)$$

Spindle Geometry

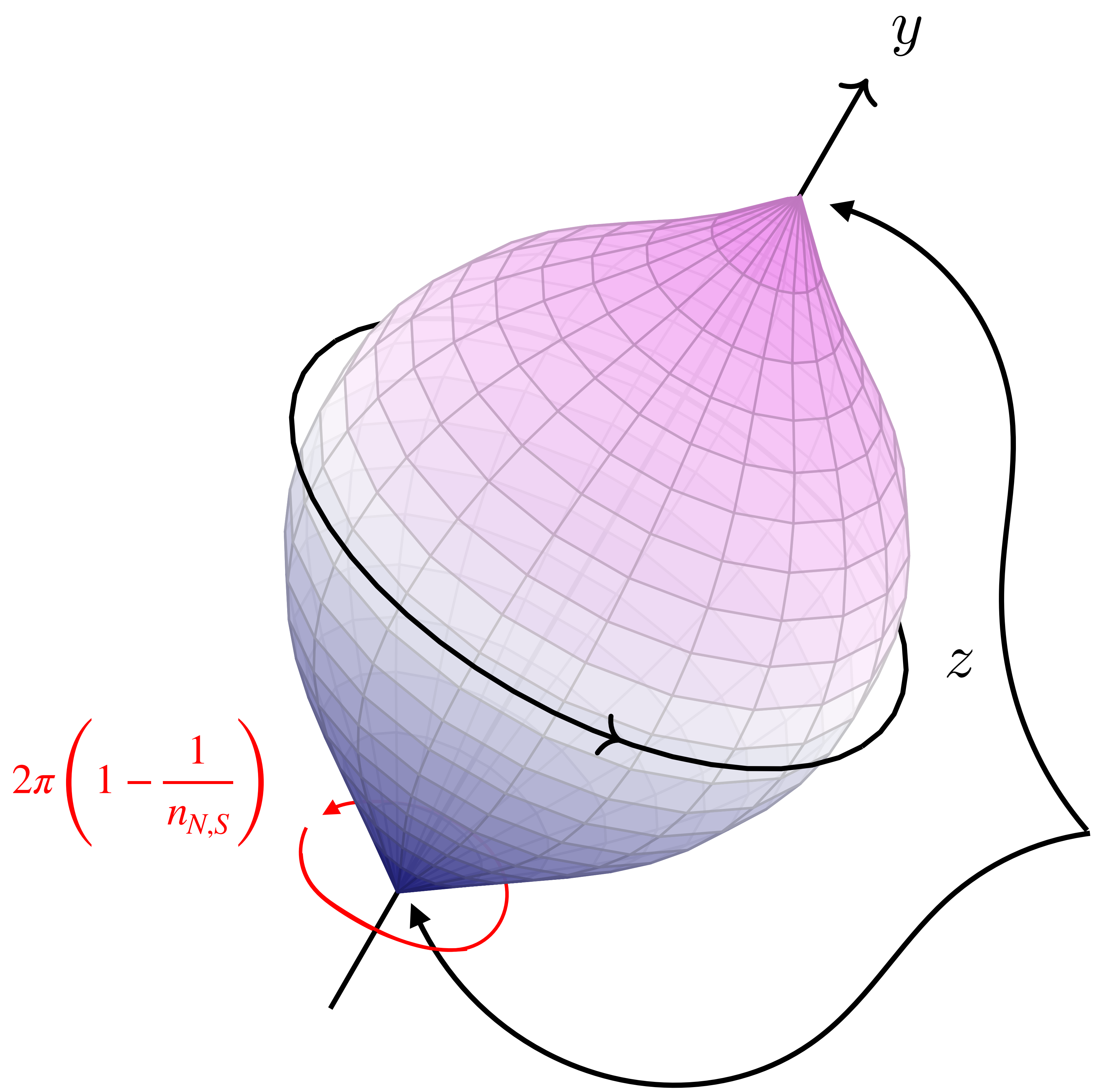


More general solutions Σ is not compact manifold, but **orbifold**. The spindle is one such geometry where SUSY is preserved

[Ferrero, Gauntlett, Ipina, Martelli, Sparks (2020, 2021)]

Spindle: topologically S^2 with conical deficit angles at poles

Spindle Geometry



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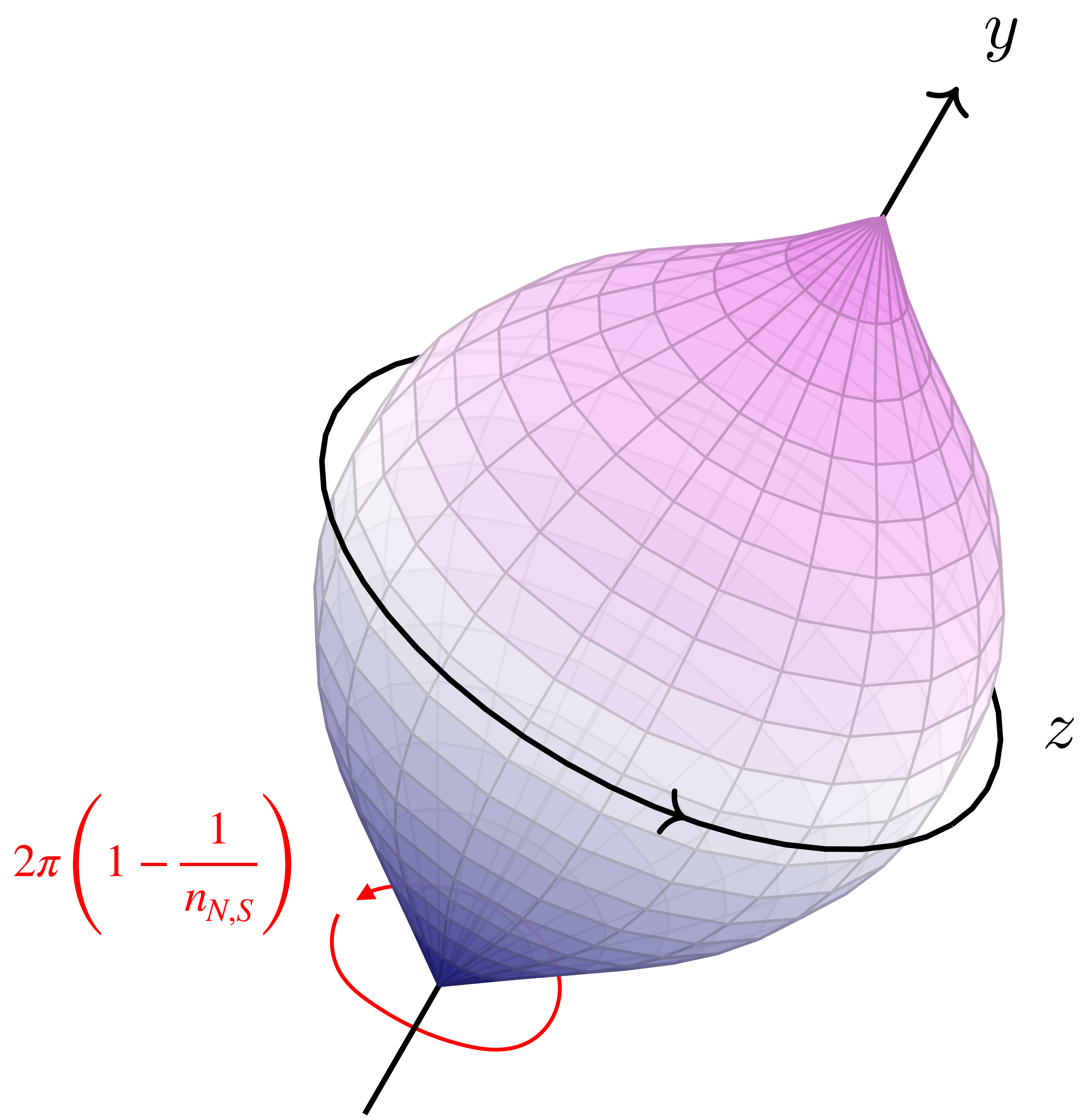

SUSY preserved in **novel** way

- Twist ($+$)
- Anti-twist ($-$)

$$\frac{1}{2\pi}\int_{\mathbb{WCP}^1_{[n_N,n_S]}} F^R = \frac{n_N \pm n_S}{n_N n_S}$$

Preserved Killing spinors

- Depend on (some) coordinates of Spindle
- Have definite chirality only at the poles

The M5 world-volume theory

The world-volume theory of an M5-brane is a $6d\ \mathcal{N} = (2{,}0)$ SCFT. No known lagrangian formulation

From $D = 11$ $\mathrm{SO}(5)$ normal bundle to M5 couples to R-symmetry $\mathrm{Sp}(2) \simeq \mathrm{SO}(5)$.

By stacking M5-branes we get $6d\ \mathcal{N} = (2{,}0)$ G=ADE SCFT. Label coming from compactification on S^1 giving $5d\ \mathcal{N} = 2$ G-SYM

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Wrapping M5s on Σ_g

Take M5 wrap on S^1 with radius $R_6 \implies 5d\ \mathcal{N}=2$ SYM

$$\int d^5x \frac{1}{g_5^2} \mathrm{tr} F \wedge \star F + \cdots \implies g_5^2 \propto R_6$$

Compactify on another S^1 with radius $R_5 \implies 4d\ \mathcal{N}=4$ SYM

$$\int dx_5 \int d^4x \frac{1}{g_5^2} \mathrm{tr} F \wedge \star F + \cdots \implies g_5^{-2} dx_5 = g_4^{-2} \implies \frac{1}{g_4^2} \sim \frac{R_5}{R_6}$$

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Wrapping M5s on Σ_g

Upshot: M5 wrapped on $T^2 \implies 4d\ \mathcal{N} = 4$ SYM w/ $g_4^{-2} \sim R_5/R_6$

Wrapping M5s on Σ_g

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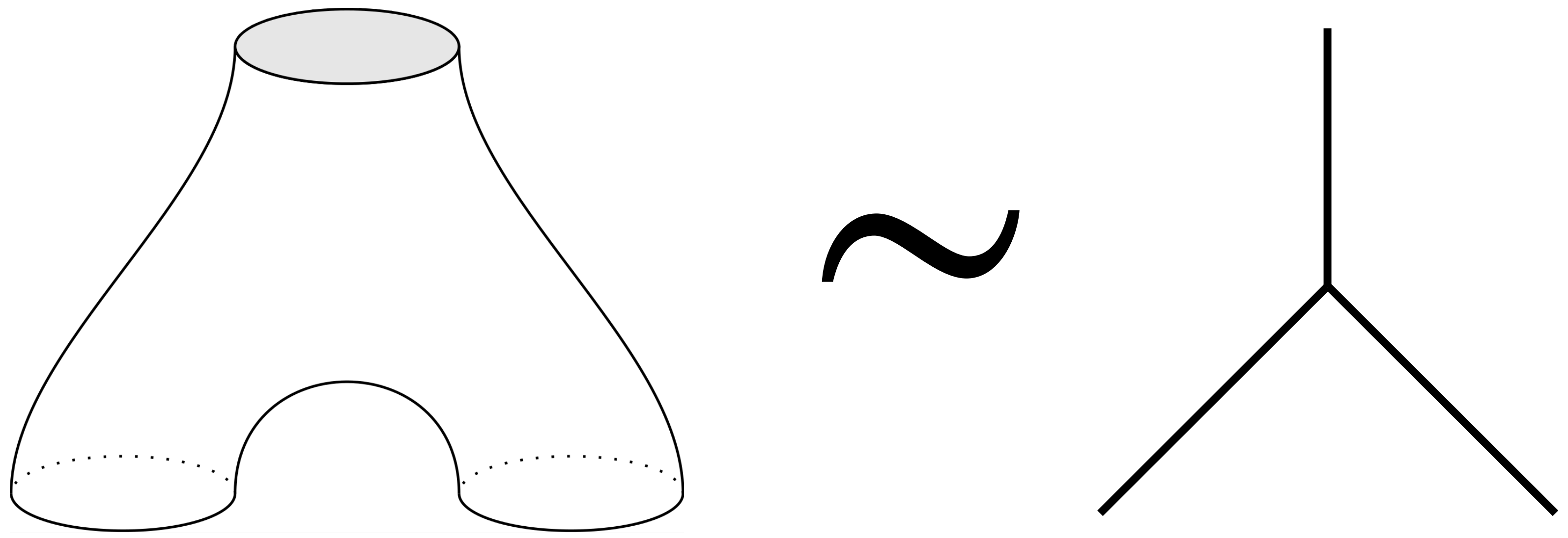
We can generalize for any (punctured) Riemann surfaces $\Sigma_{g,n}$: **class-S** theories [Gaiotto (2009)]

Wrapping M5s on Σ_g

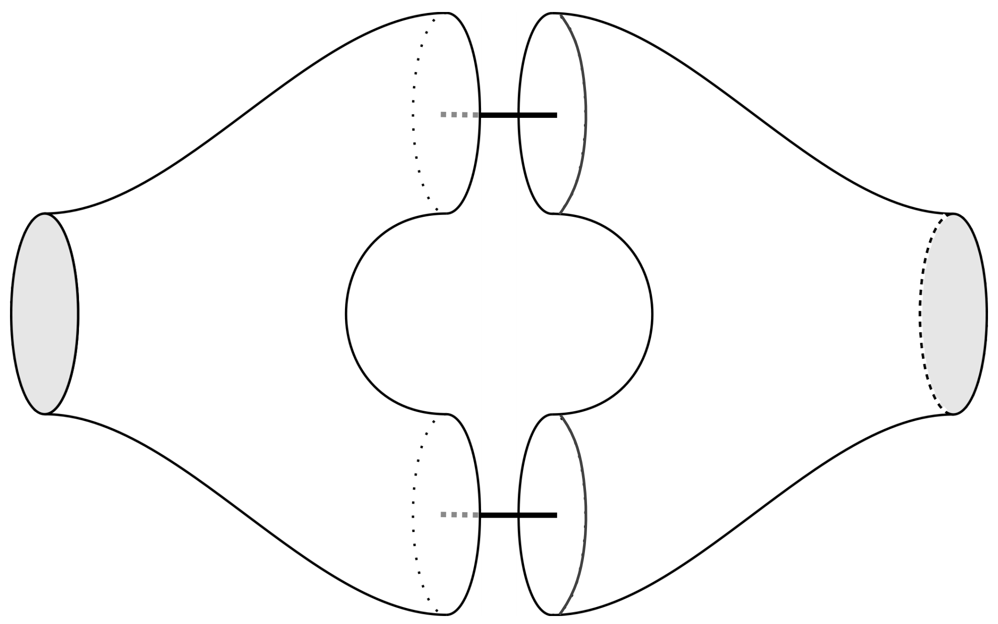
Any Riemann surface can be decomposed into pair of pants

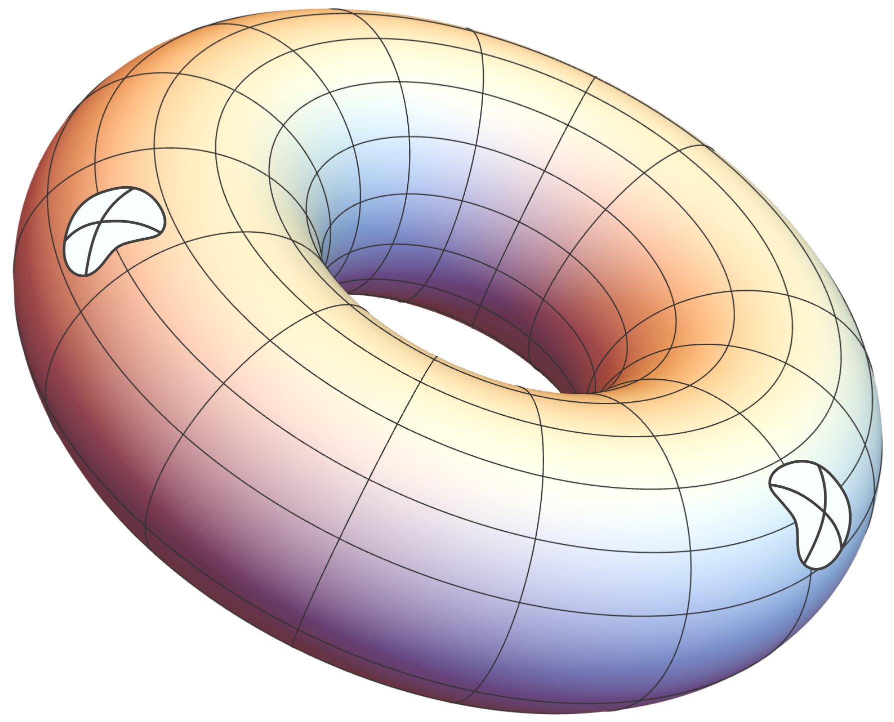


T_N block : $\mathcal{N} = 2$ SCFTs with $\mathrm{SU}(2) \times \mathrm{U}(1)_R \times \mathrm{SU}(N)^3$ global symmetry as world-volume theories of stack of M5 on three-punctured sphere.

Wrapping M5s on Σ_g

Gluing T_N blocks is gauging some $\mathrm{SU}(N)$: higher genus Riemann surfaces





$\Sigma_{1,2}$

S-class: gluing with $\mathcal{N} = 2$ vector multiplet

The B3W Model

Up to now, compactification on Riemann surface. Generalization to wrapping branes on **calibrated cycles on CYs.** Calibration needed for twisting, aka preserve SUSY

The B3W Model

Further generalization: CY_3 as holo $\mathbb{C}^2$ bundle over C_g

[Bah, Beem, Bobev, Wecht (2012)]

$$\begin{array}{ccc} \mathbb{C}^2 & \hookrightarrow & CY_3 \\ & & \downarrow{\scriptstyle \pi} \\ & & C_g \end{array}$$

$$CY_3 = K_{C_g} \otimes V$$

To preserve SUSY

$$\begin{array}{ccc} \mathrm{SU}(2) & \longrightarrow & V \\ & & \downarrow{\scriptstyle \pi} \\ & & C_g \end{array}$$

Central $\mathrm{U}(1)$ in $\mathrm{U}(2)$ connection on CY constrained $\Longrightarrow$ R-symmetry

The B3W Model

Further generalization [Bah, Beem, Bobev, Wecht (2012)]:

$$\begin{tikzcd} \mathrm{SU}(2) \arrow[r] & V \arrow[d, "\pi"] \\ & C_g \end{tikzcd}$$

IR dynamics of branes wrapped on this geometry depend on **choice of this bundle**

The B3W Model

When $\mathrm{CY}_3 = \mathscr{L}_1 \oplus \mathscr{L}_2$ (decomposable)

Family of IR SCFT described by degree of line bundles

$$c_1(\mathscr{L}_1) = p,\ c_1(\mathscr{L}_2) = q,\ p + q = 2g - 2$$

$$\begin{tikzcd} \mathbb{C}^2 \arrow[r, hook] & \mathcal{L}_1 \oplus \mathcal{L}_2 \arrow[d, "\pi"] \\ & C_g \end{tikzcd}$$

Manifest $\mathrm{U}(1)^2$ isometry $\implies \mathrm{U}(1)_R \times \mathrm{U}(1)_F$

Reparametrization $p = (1 + \mathbf{z})(g - 1), \quad q = (1 - \mathbf{z})(g - 1)$ where $\mathbf{z}(g - 1) \in \mathbb{Z}$

The B3W Model

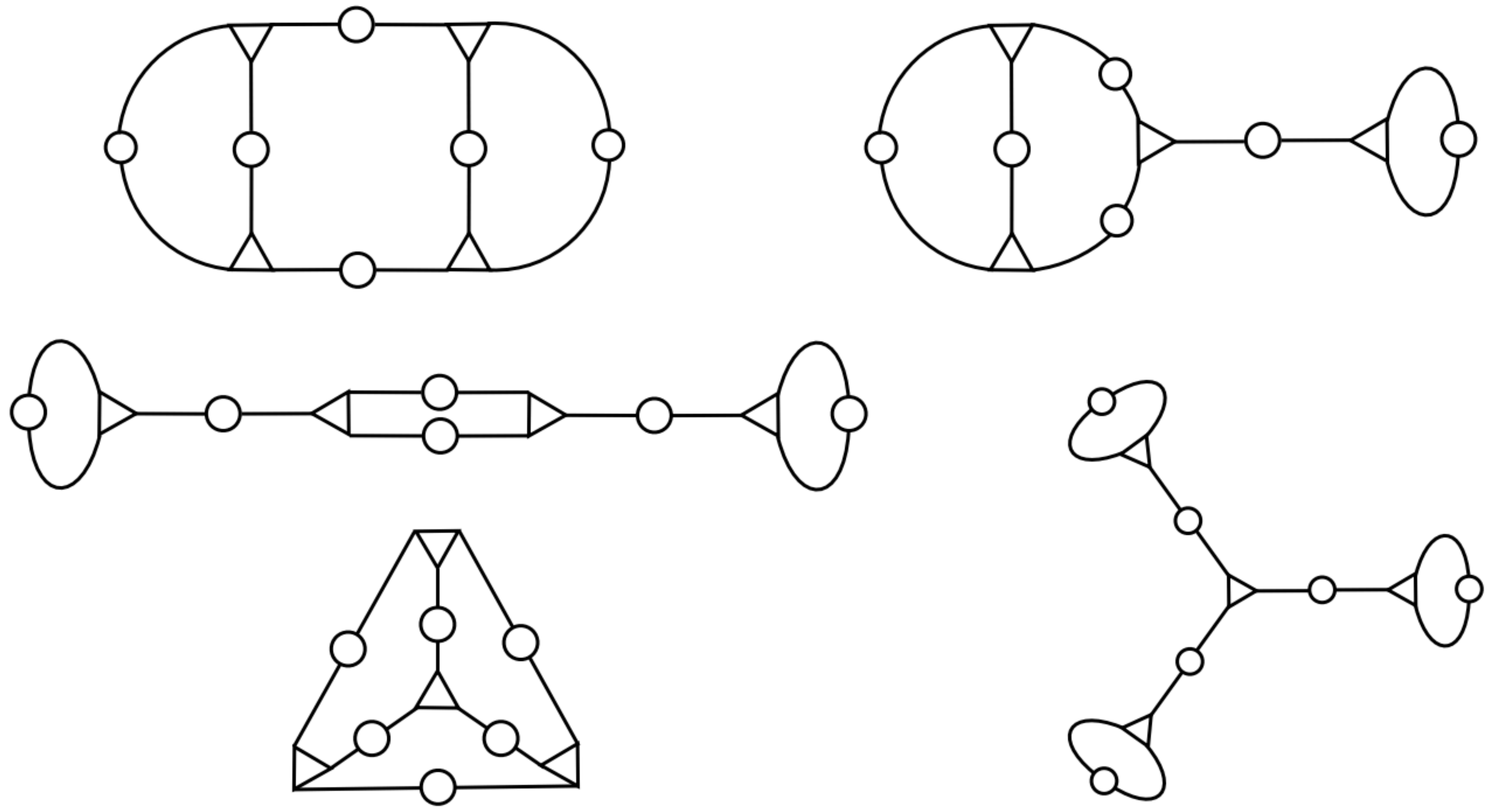
Limiting cases $\quad c_1(\mathscr{L}_1) = p,\ c_1(\mathscr{L}_2) = q,\ p + q = 2g - 2$

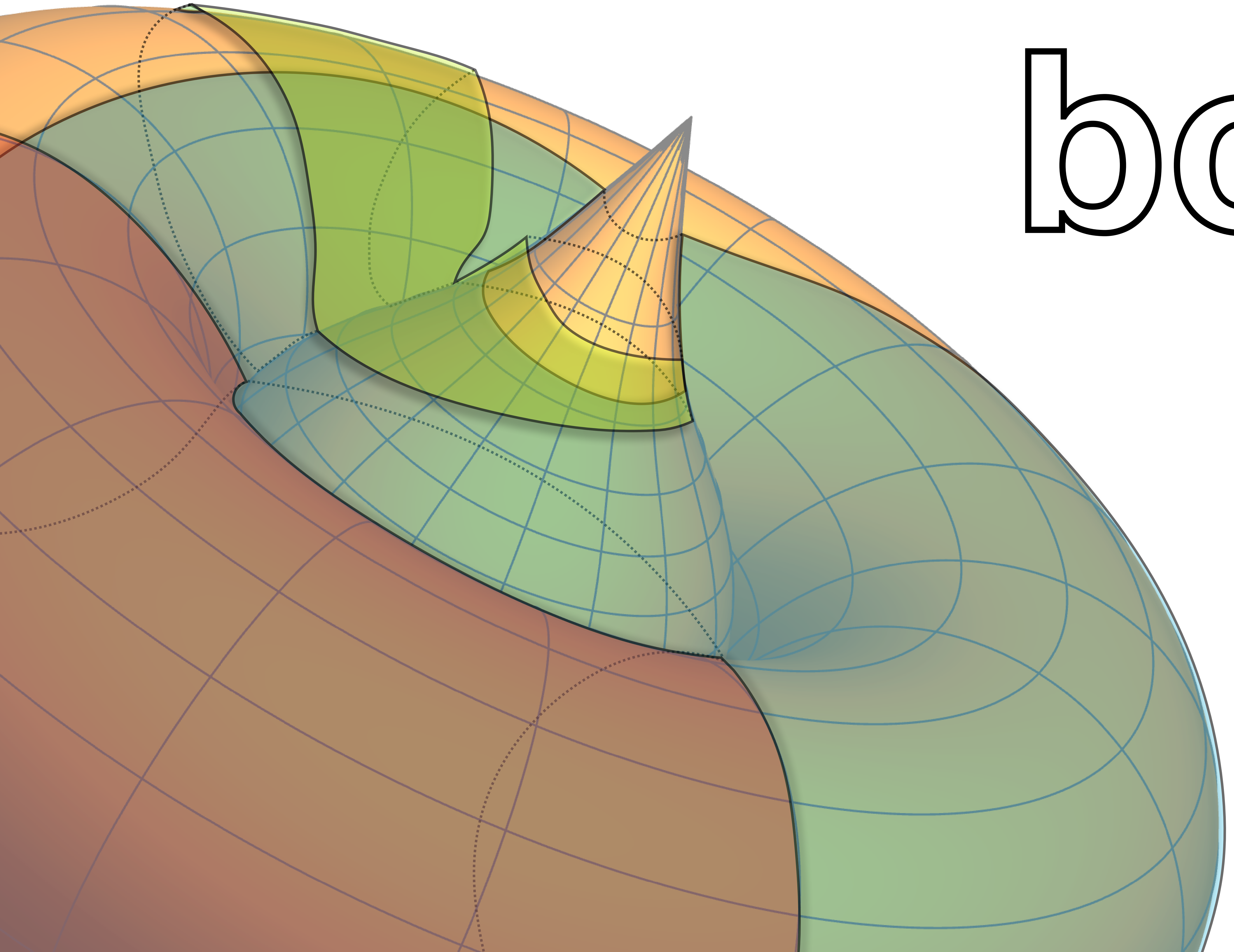
$q = 0$ or $p = 0 \implies X = \mathbb{C} \times T^\star C_g,\ \mathcal{N} = 2$ S-class [Gaiotto (2009)]

$q = p,\ \mathcal{N} = 1$ Sicilian gauge theories [Benini, Tachikawa, Wecht (2009)]

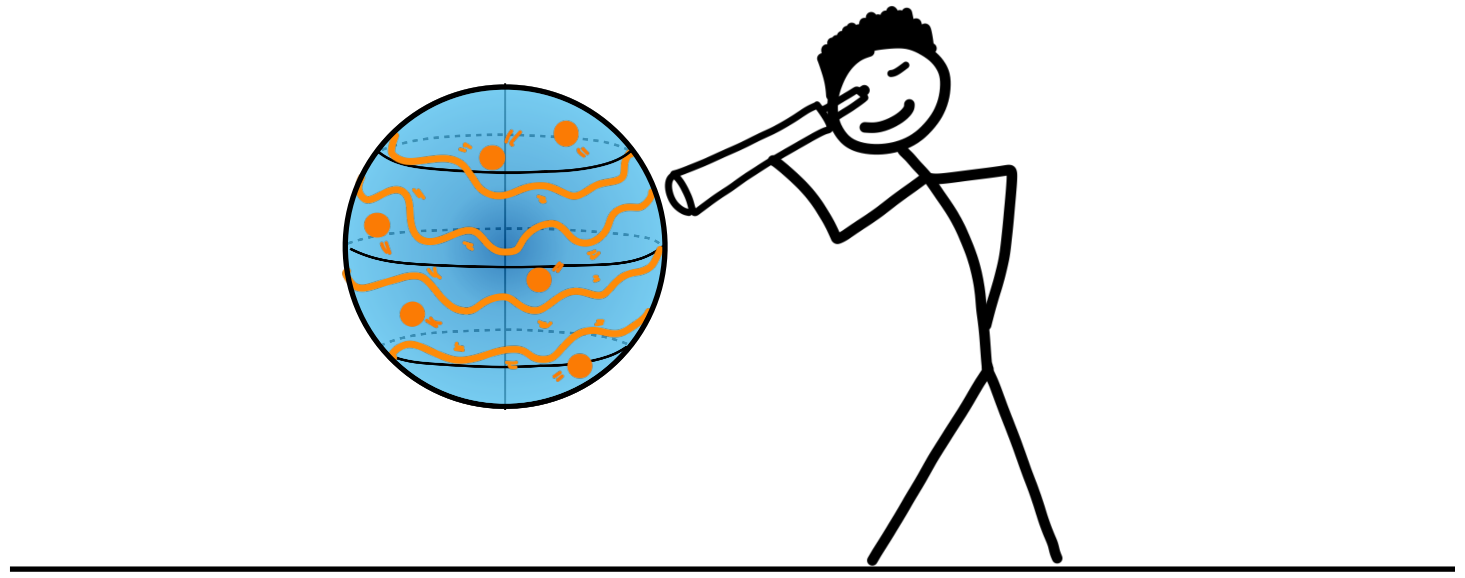
The B3W Model

General p, q can be constructed from opportune gluing of $2(g-1)$ T_N blocks to form a Riemann surface with no punctures. Gluing with both $\mathcal{N}=1{,}2$ vector multiplets $\Longrightarrow$ choice of p, q





The boundary



Outline

General introduction

The boundary

- The anomaly polynomial of an M5-brane
- Stacking the branes
- Wrapping the branes
- Two-dimensional central charge

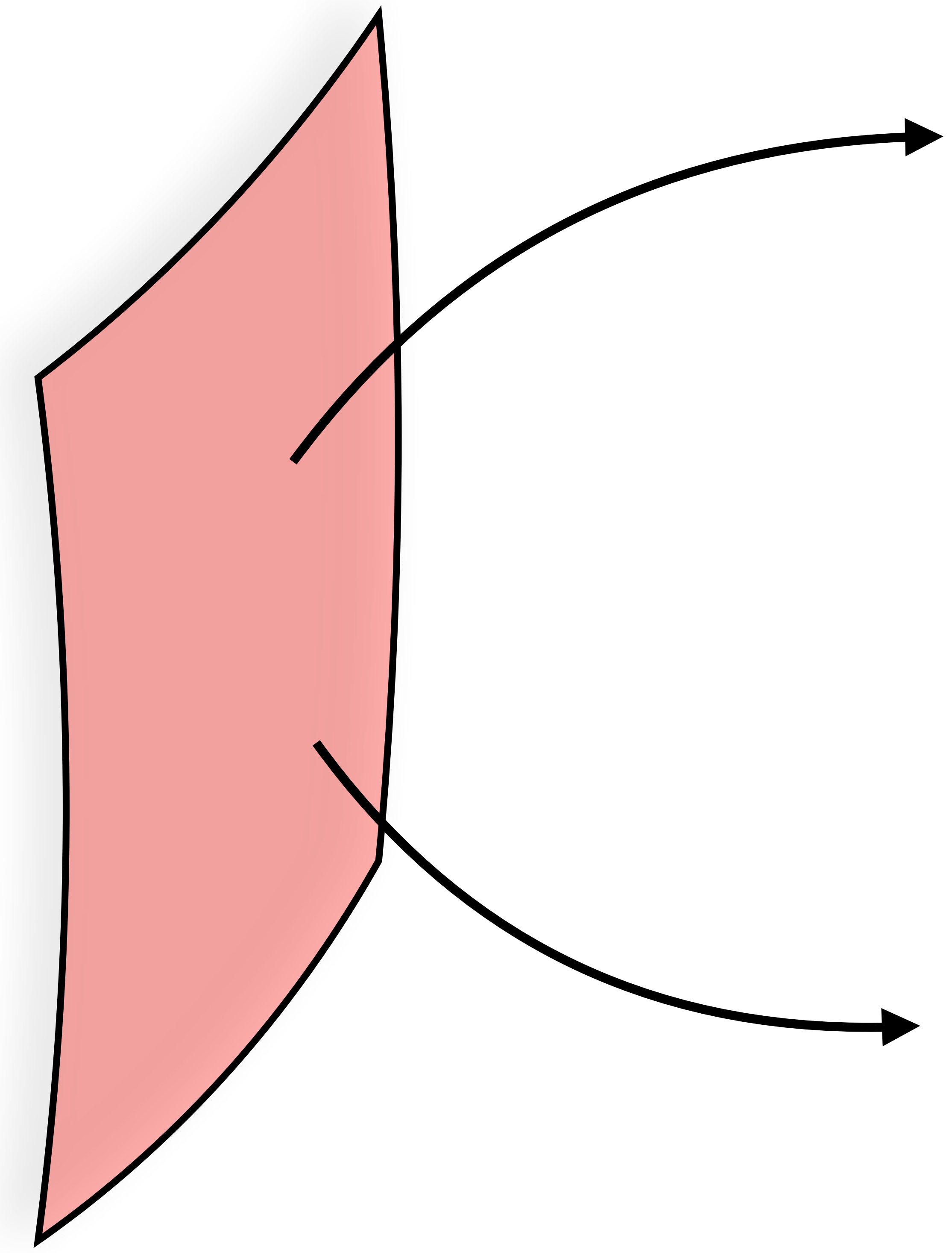
The bulk

Anomaly Polynomial of M5-brane

Supersymmetric $\mathcal{N} = (2,0)$ abelian tensor multiplet in $6d$: [Witten (1996)]

- Self-dual three form
- 5 scalars
- 4 real Weyl fermions

Anomaly Polynomial of M5-brane



Four components chiral spinors

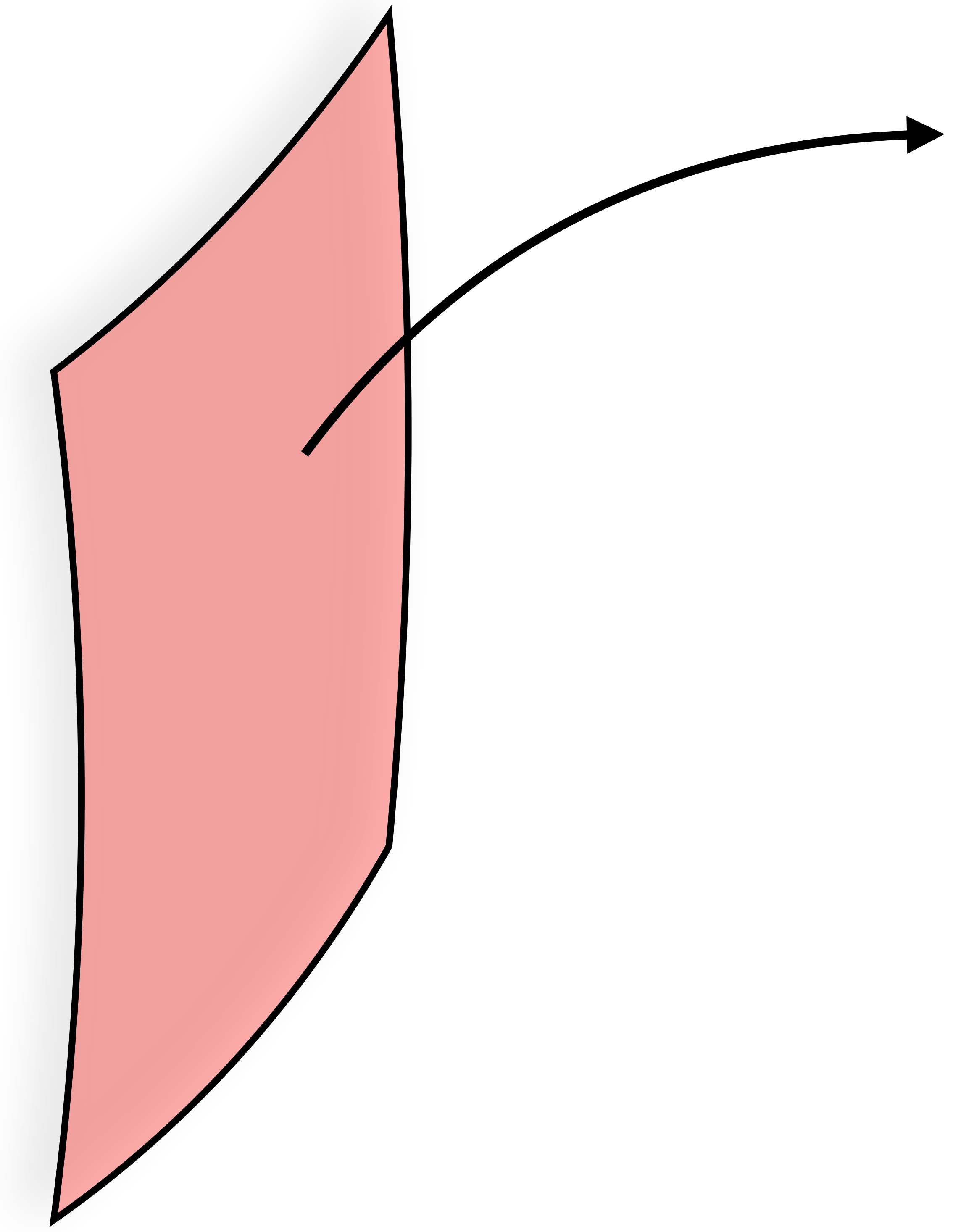
$$I_D = \frac{1}{2} \mathrm{ch} S(N) \hat{A}(TW)$$

Self-dual chiral two-form

$$I_A = \frac{1}{5760} \left(16 p_1(TW)^2 - 112 p_2(TW)\right)$$

[Witten, Gaume (1986)]

Anomaly Polynomial of M5-brane



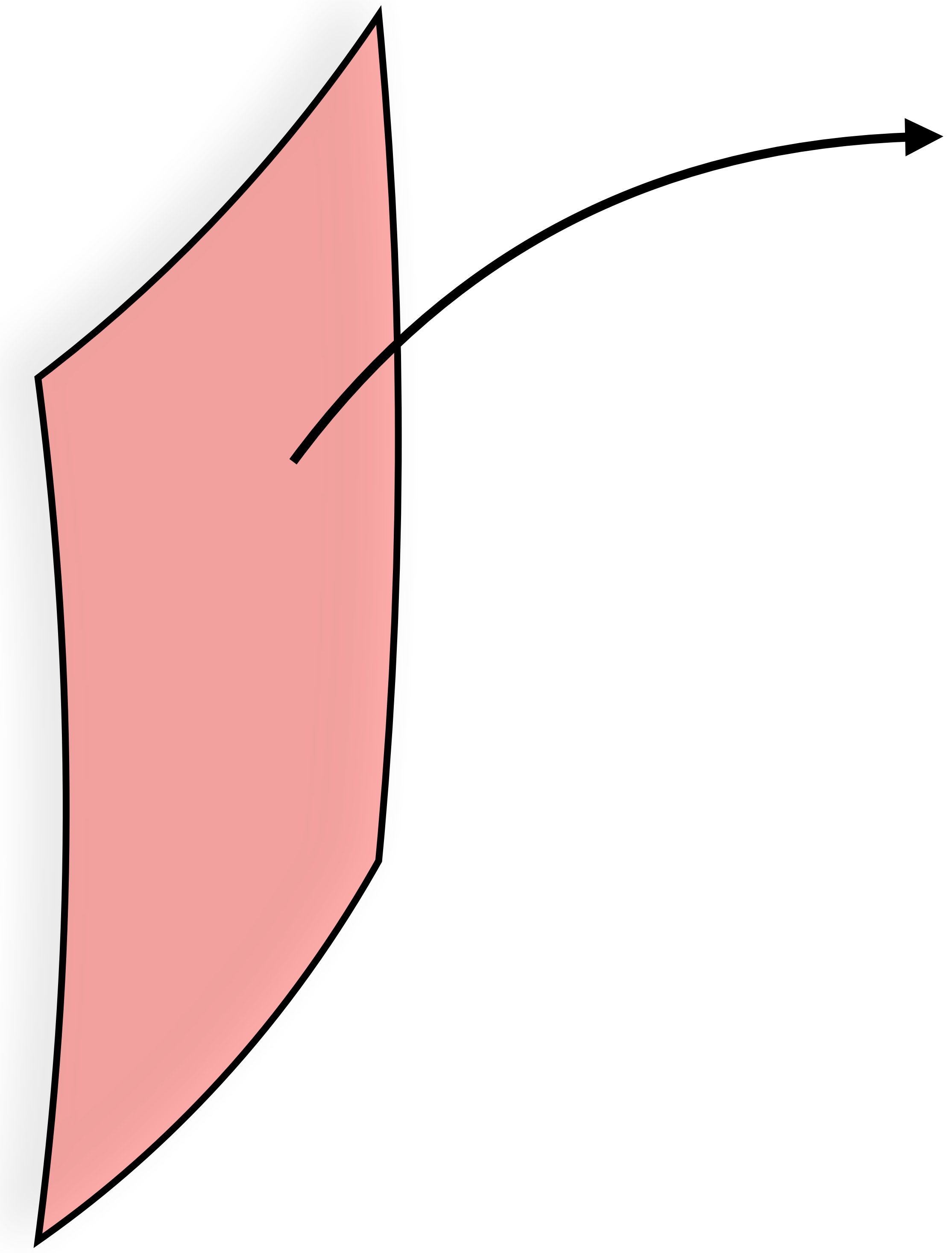
Four components chiral spinors

$$I_D = \frac{1}{2}\mathrm{ch}S(N)\,\hat{A}(TW)$$

Sections of rank-four spinor bundle constructed from the normal bundle N using the spinor rep of $SO(5)$

$SO(5)$ is the remaining isometry from M-theory after M5 defect insertion

Anomaly Polynomial of M5-brane

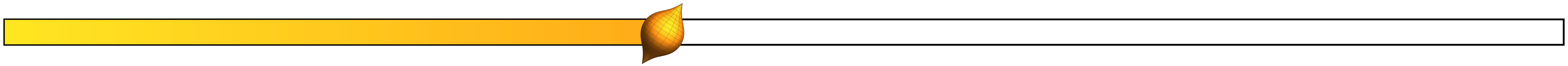


Four components chiral spinors

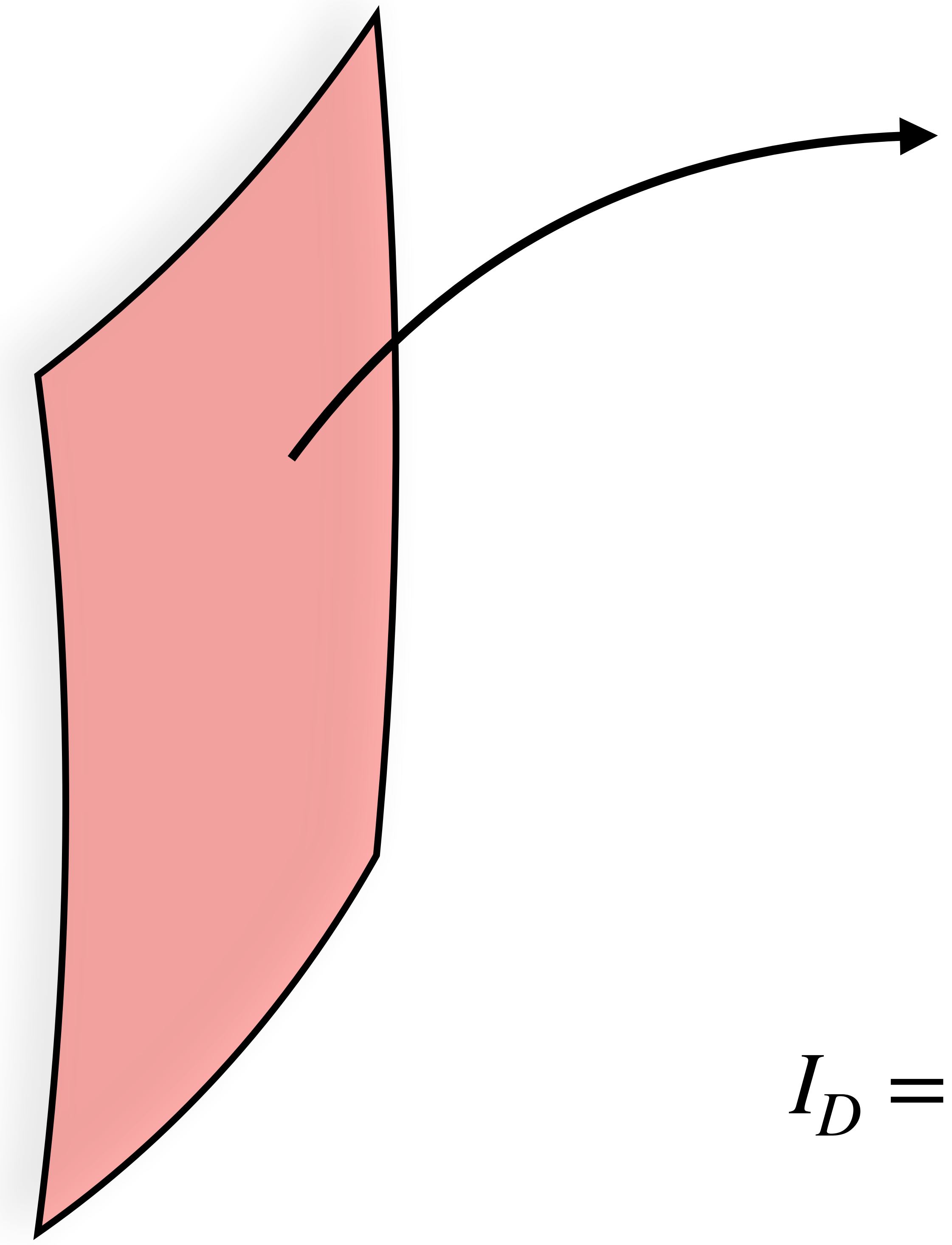
$$I_D = \frac{1}{2}\mathrm{ch}S(N)\,\hat{A}(TW)$$

$\hat{A}(TW)$ is the Dirac genus of TW, index of the Dirac operator on it

$$\hat{A}(TW) = 1 - \frac{p_1(TW)}{24} + \frac{7p_1(TW)^2 - 4p_2(TW)}{5760}$$



Anomaly Polynomial of M5-brane

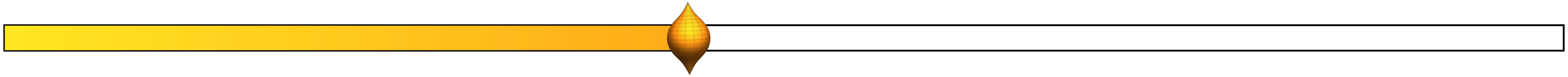


Four components chiral spinors

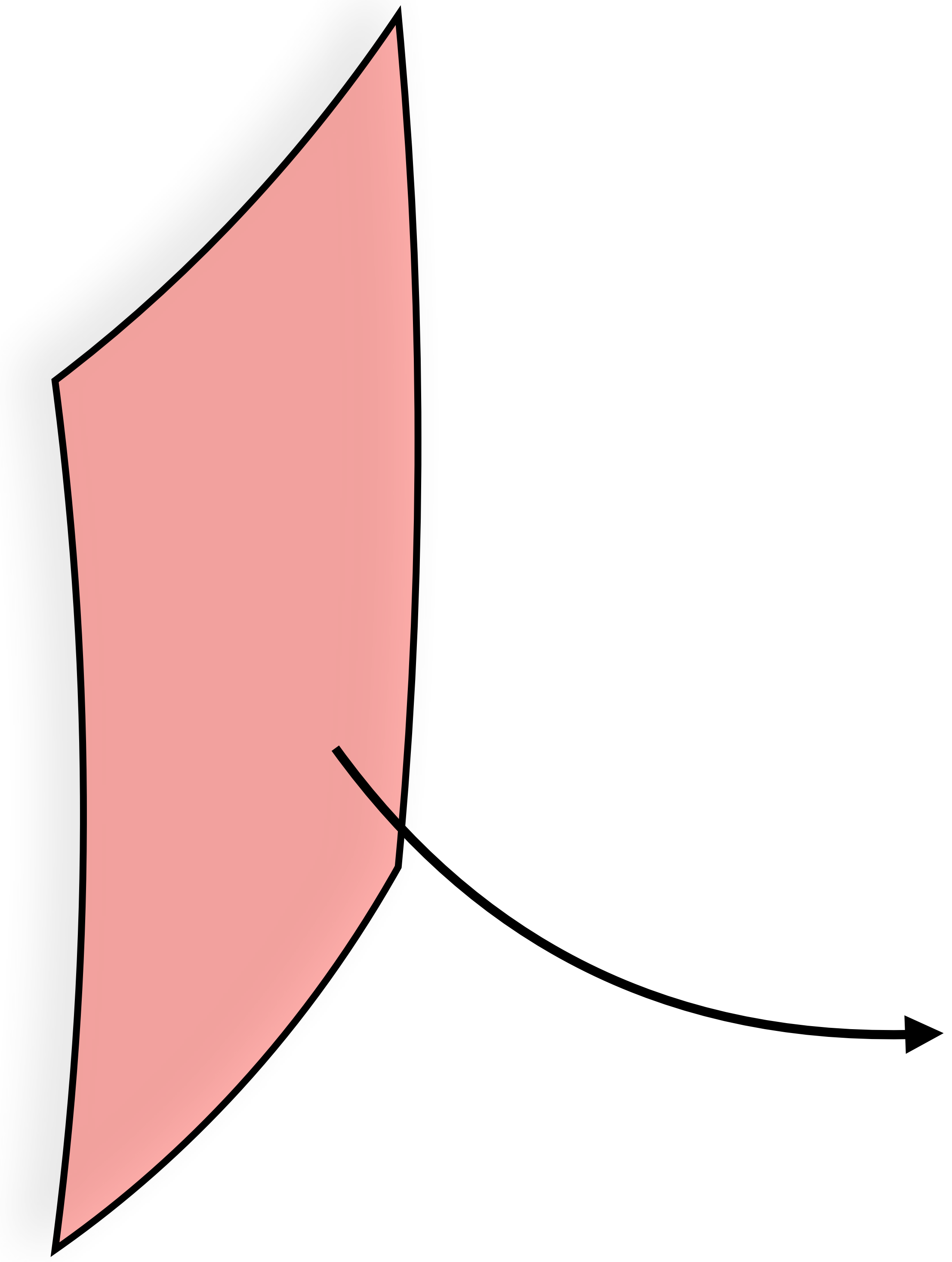
$$I_D = \frac{1}{2}\mathrm{ch}S(N)\,\hat{A}(TW)$$

$$\Downarrow$$

$$I_D = \frac{1}{2}\left(\frac{p_2(N)^2}{24} + \frac{p_1(N)^2}{96} - \frac{p_1(N)p_1(TW)}{48} + \frac{7p_1(TW)^2 - 4p_2(TW)}{1440}\right)$$



Anomaly Polynomial of M5-brane

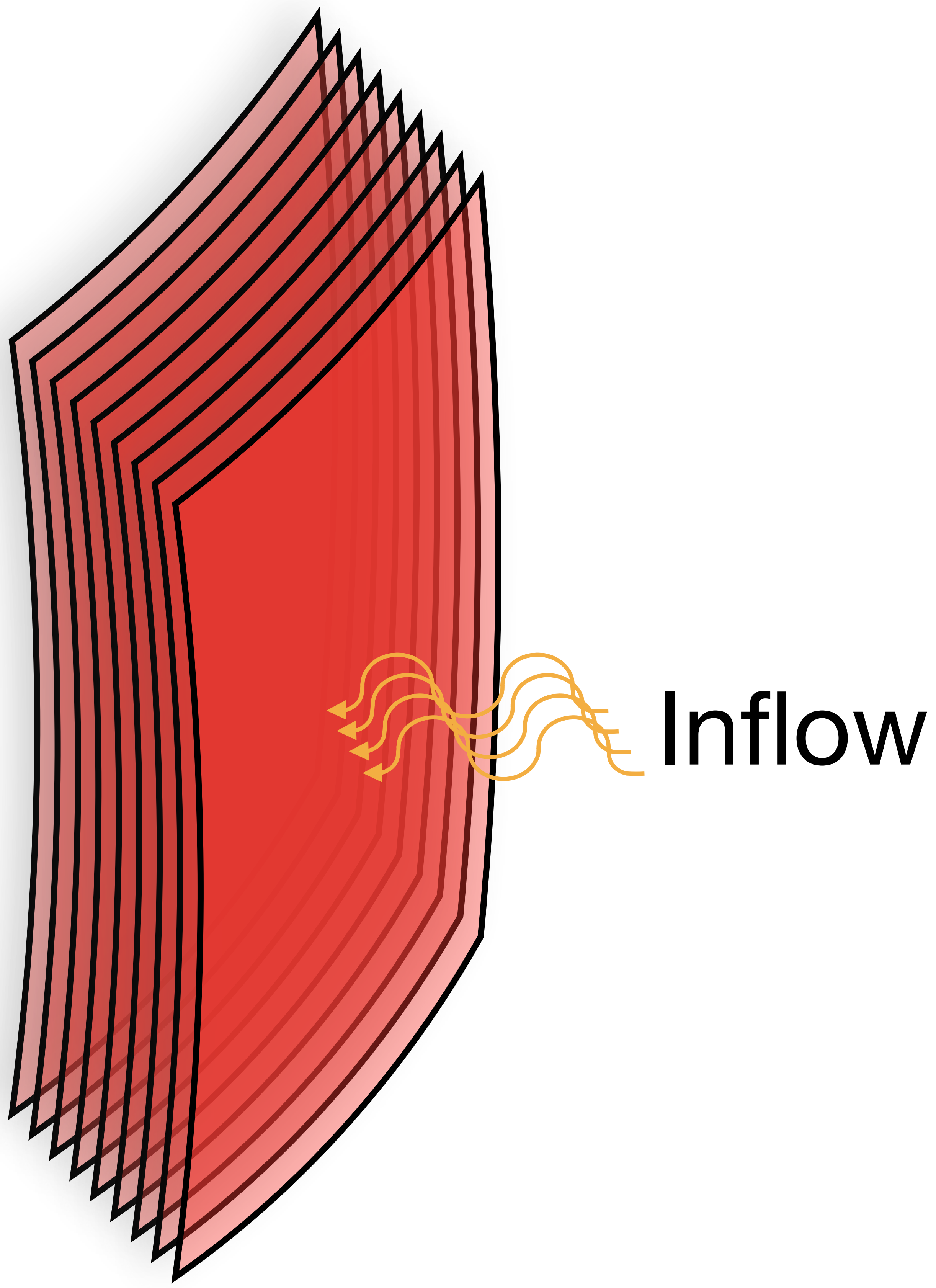


Propagates on brane world-volume W,
does not see normal bundle

Self-dual chiral two-form

$$I_A = \frac{1}{5760}\left(16p_1(TW)^2 - 112p_2(TW)\right)$$

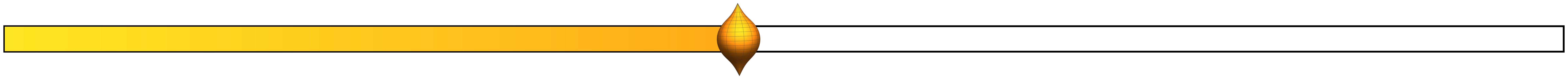
Stacking the Branes



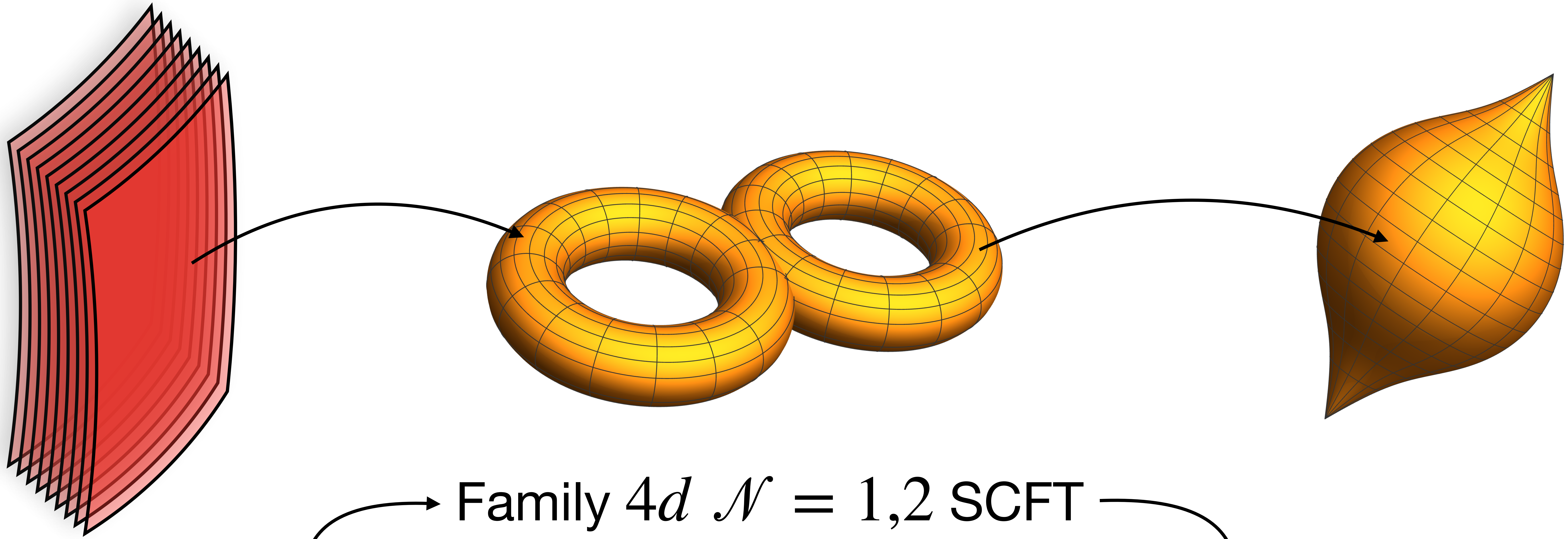
Anomaly polynomial

$$I_8 = \underbrace{\frac{N-1}{48}\left[p_2(NW) - p_2(TW) + \frac{1}{4}(p_1(TW) - p_1(NW))^2\right]}_{\text{Spinors+Three-form}} + \underbrace{\frac{N^3-N}{24}p_2(NW)}_{\text{CS term}}$$

[Witten (1996)] [Harvey, Minasian, Moore (1998)]
[Intriligator (2000)] [Yi (2001)] …



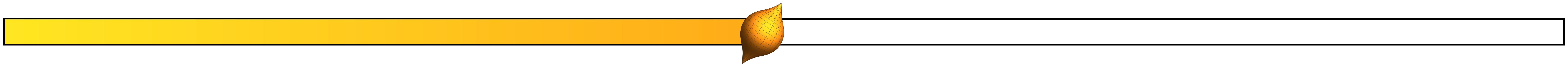
Wrapping the Branes



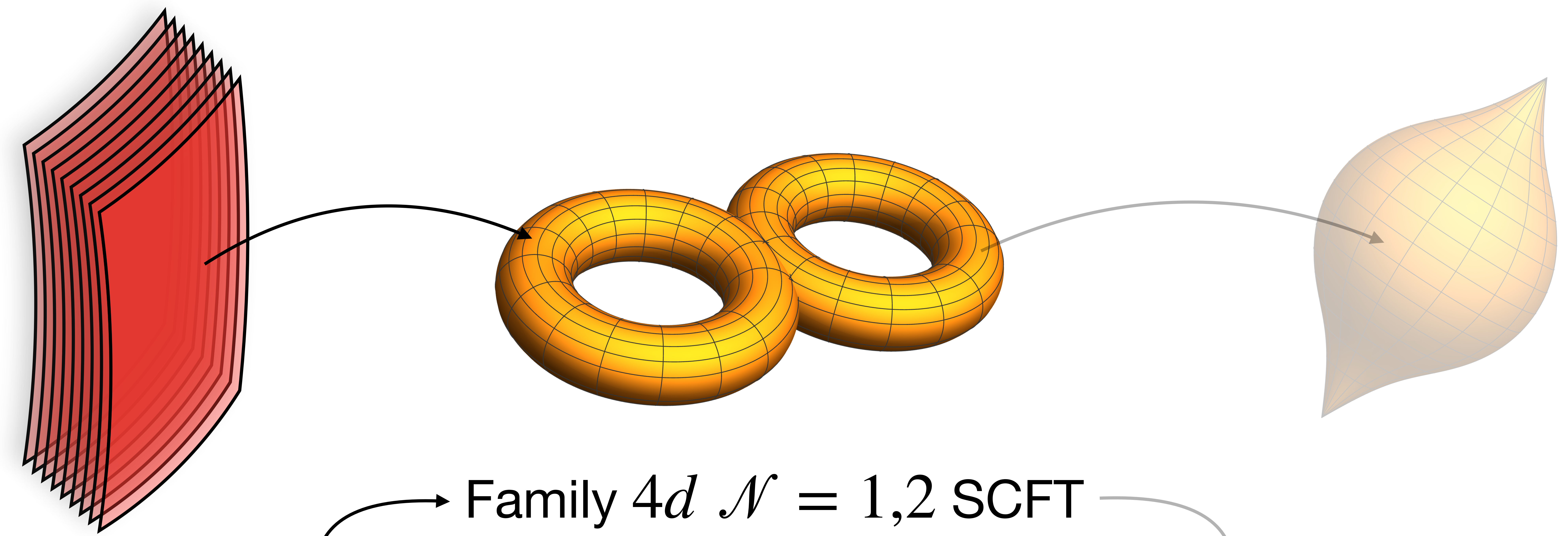
Family $4d$ $\mathcal{N} = 1{,}2$ SCFT

$6d$ $\mathcal{N} = (2{,}0)$ A_{N-1} SCFT

Family $2d$ $\mathcal{N} = (2{,}0)$ SCFT



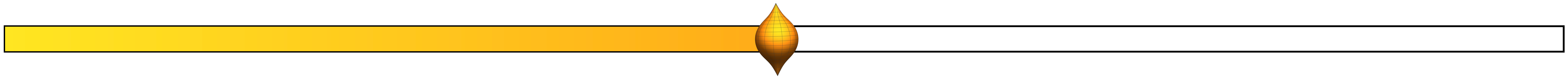
Wrapping the Branes



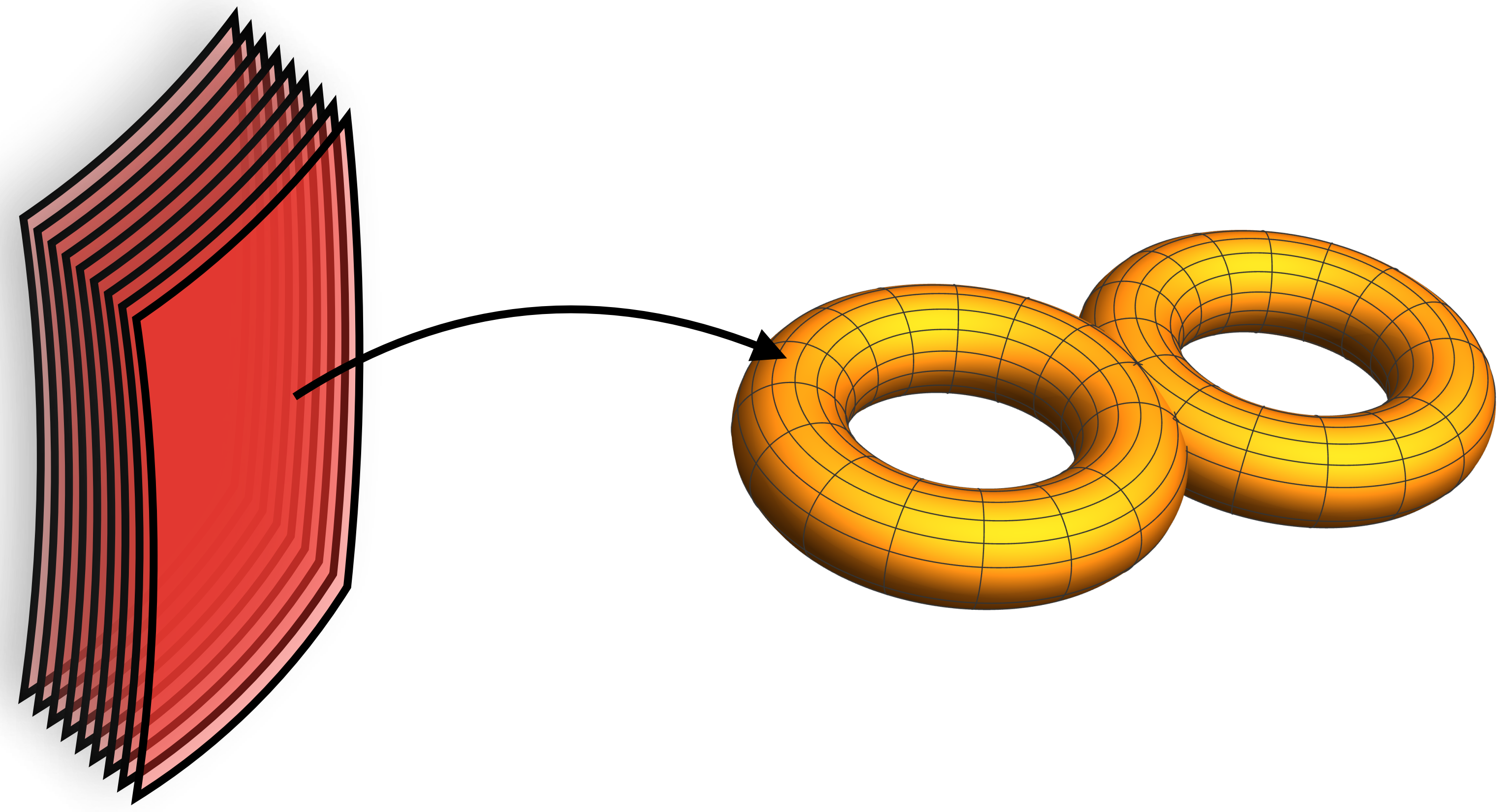
Family $4d$ $\mathcal{N} = 1{,}2$ SCFT

$6d$ $\mathcal{N} = (2{,}0)$ A_{N-1} SCFT

Family $2d$ $\mathcal{N} = (2{,}0)$ SCFT



Wrapping the Branes



$$I_6 = \int_{C_g} I_8 \sim \sum_{i,j,k=1,2} A_{ijk} c_1(F_i) c_1(F_j) c_1(F_k)$$

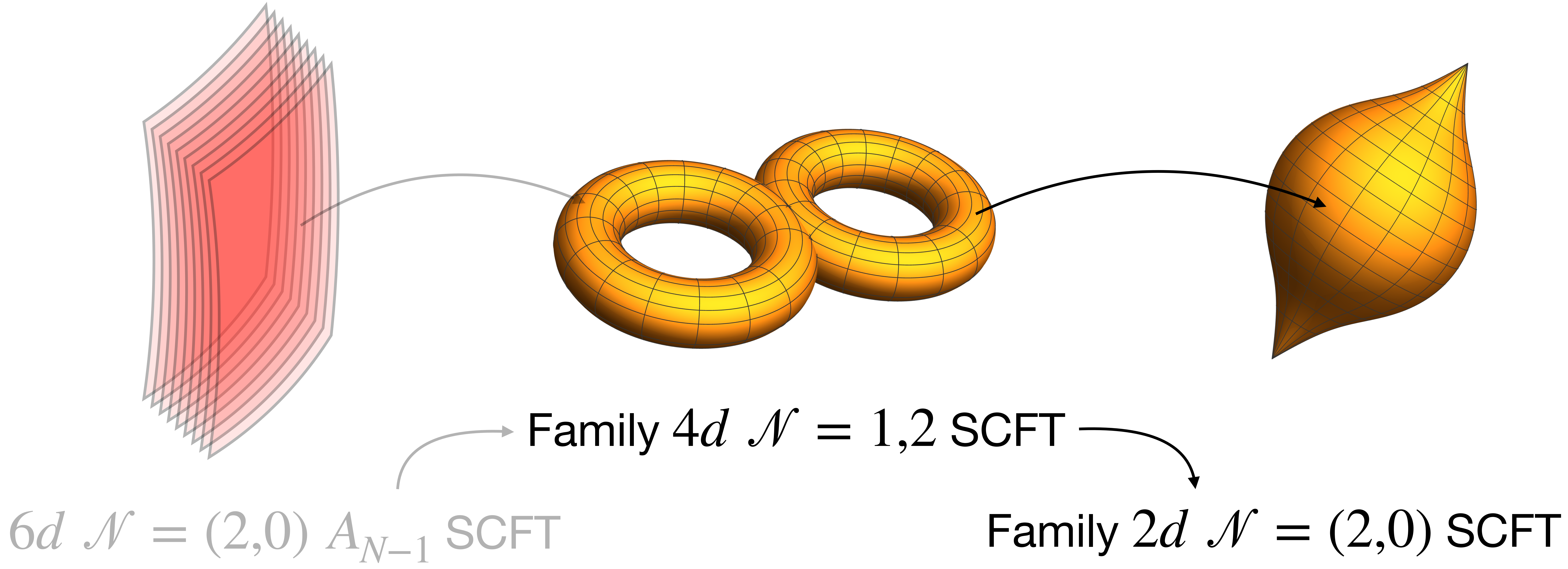
Where A_{ijk} abelian anomalies of $4d$ theory

$$A_{RRR} = (g-1)N^3 \qquad A_{RRF} = -\frac{1}{3}(g-1)zN^3$$

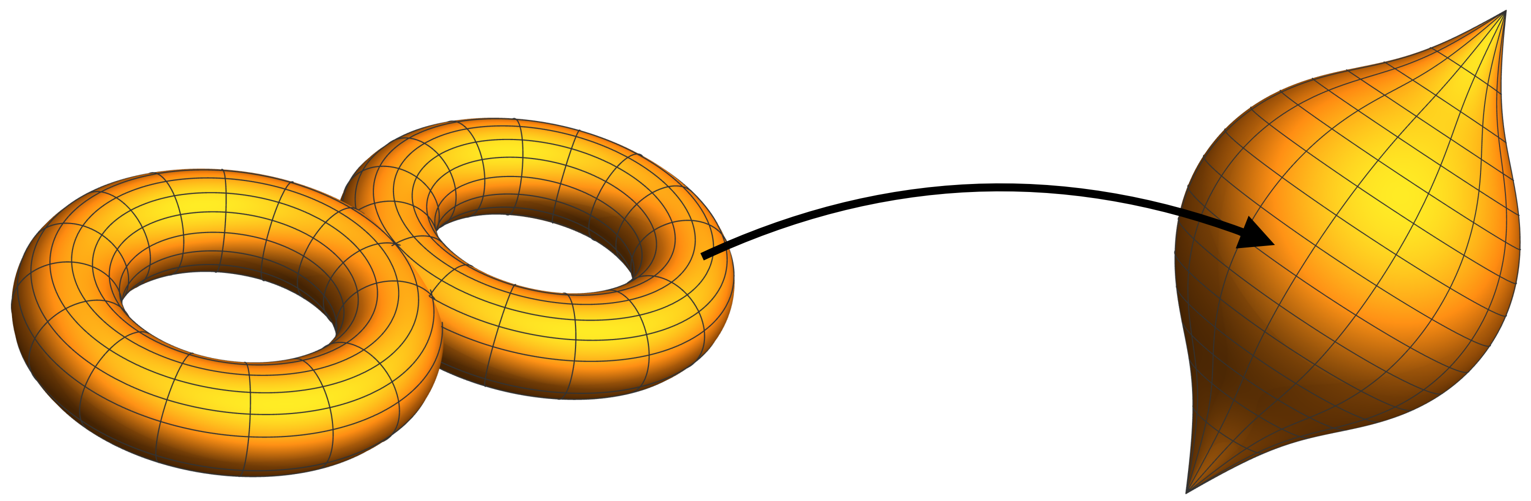
$$A_{RFF} = -\frac{1}{3}(g-1)N^3 \qquad A_{FFF} = (g-1)zN^3$$

[Bah, Beem, Bobev, Wecht (2012)]

Wrapping the Branes



Two-dimensional Central Charge



[Amariti, Mancani, Morgante, Petri, Segati (2023)]

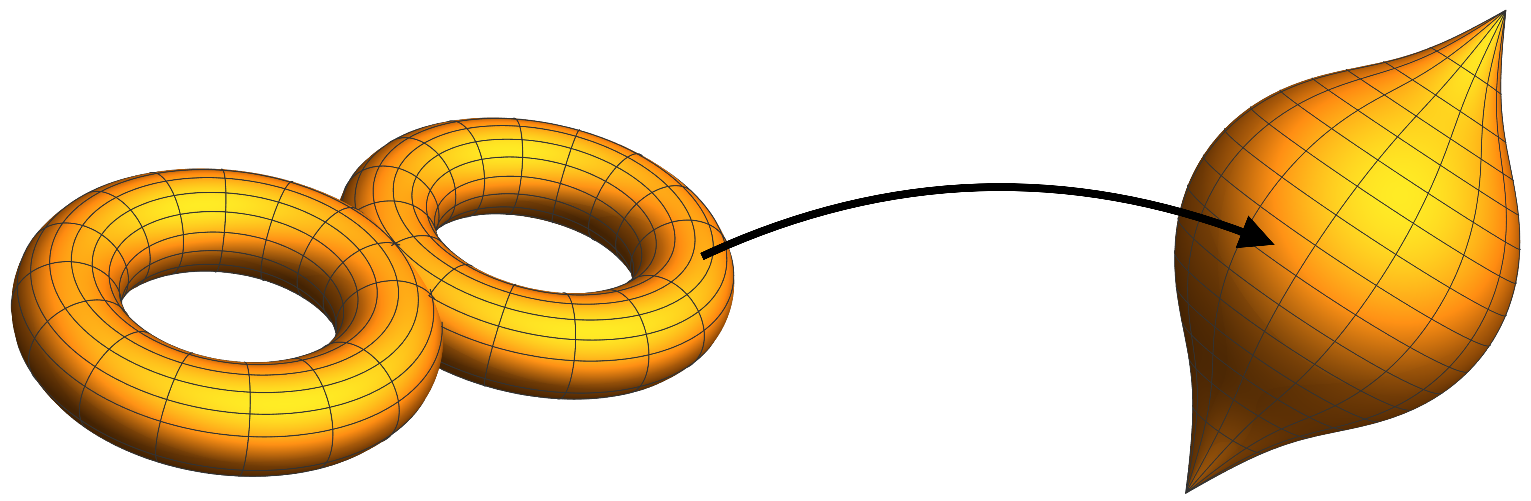
Additional abelian symmetry from azimuthal rotation on spindle

$$I_4 = \int_{\mathbb{WCP}^1_{[n_N,n_S]}} I_6$$

Subleading at large N

$$\sim \frac{c_R}{6} c_1(F_R)^2 \cancel{- \frac{c_R - c_L}{24} p_1(TW_2)}$$

Two-dimensional Central Charge



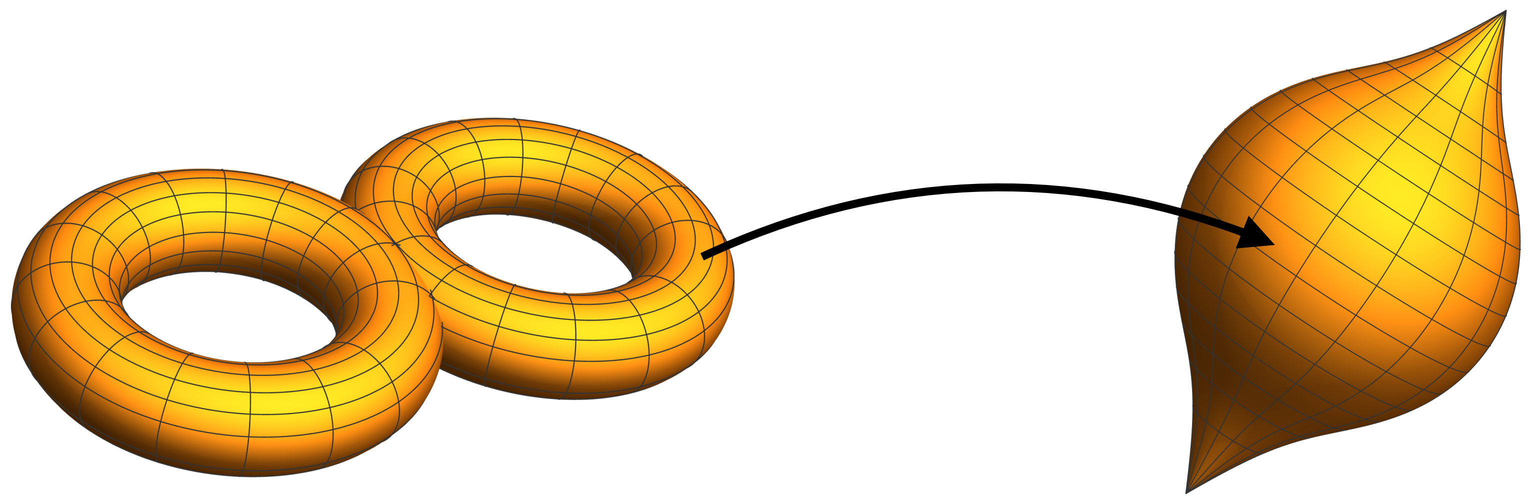
[Amariti, Mancani, Morgante, Petri, Segati (2023)]

Fix magnetic fluxes

$$\int c_1(F_R) = \frac{p_R}{n_N n_S} \qquad \int c_1(F_F) = \frac{p_F}{n_N n_S}$$

Preserve SUSY by R-symmetry (anti-)twist

Two-dimensional Central Charge



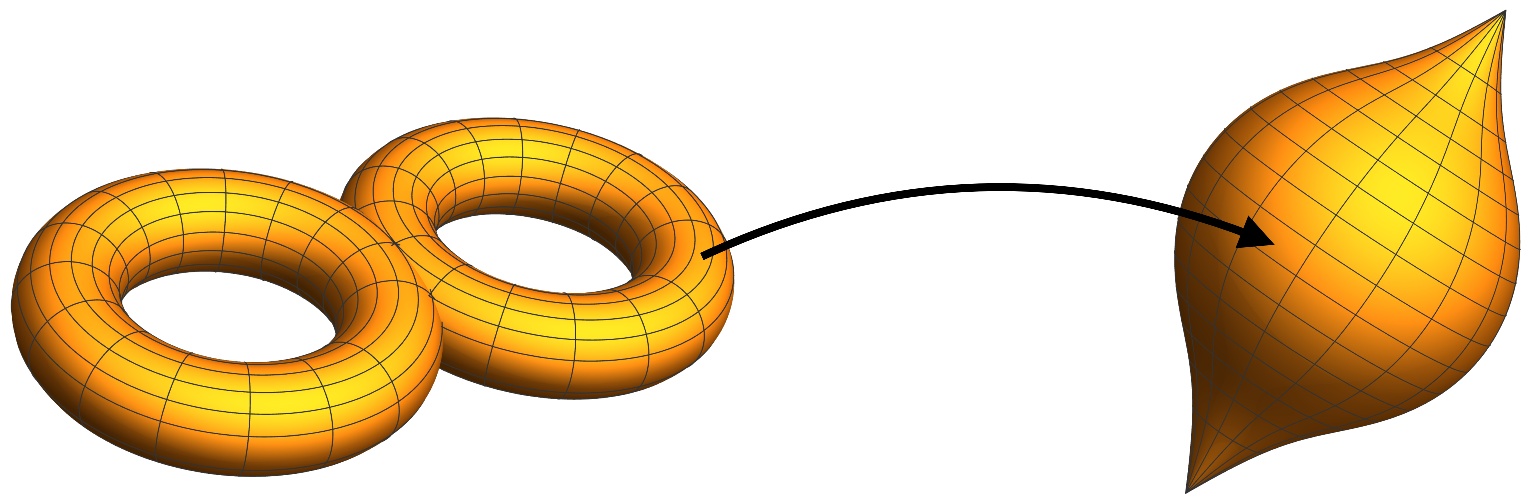
[Amariti, Mancani, Morgante, Petri, Segati (2023)]

$$\rho_R(y_N) = \frac{(-1)^{t_N}}{n_N} \qquad \rho_R(y_S) = \frac{(-1)^{t_S+1}}{n_S}$$

Where $t_N = 0,1$. Twist $t_S = t_N$, anti-twist $t_S = t_N + 1$

Flavour flux fixed up to arbitrary constant

Two-dimensional Central Charge

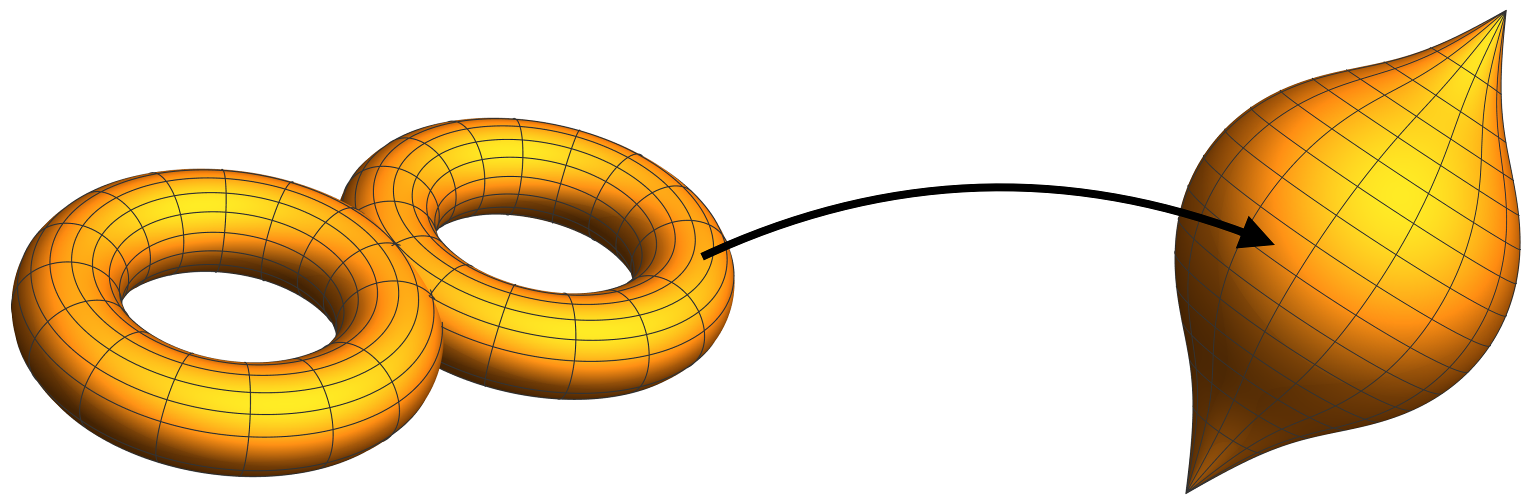


[Amariti, Mancani, Morgante, Petri, Segati (2023)]

Central charge in large-N from anomaly polynomial and allow mixing. New $\mathrm{U}(1)_J$ from azimuthal symmetry of spindle

$$R^{\mathrm{trial}}(x, \epsilon) = R_0 + xF + \epsilon J$$

Two-dimensional Central Charge



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Central charge in large-N from anomaly polynomial and allow mixing. New $\mathrm{U}(1)_J$ from azimuthal symmetry of spindle

$$R^{\mathrm{trial}}(x, \epsilon) = R_0 + xF + \epsilon J$$

$$c_R(\epsilon, x) = \frac{6 I_4(\epsilon, x)}{c_1(F_R)^2}$$

c-extremization!

Two-dimensional Central Charge

N.B. $\mathbf{z} = \dfrac{p-q}{p+q}$

Twist ($+$) and anti-twist ($-$)

$$c_{2d}^{\pm} = \frac{N^3(g-1)\left(4p_F^2 - \left(n_N \pm n_S\right)^2\right)\left(2zp_F + (-1)^{t_N}\left(n_N \pm n_S\right)\right)\left((-1)^{t_N}\left(n_N \pm n_S\right)\left(16zp_F + \left(z^2+3\right)(-1)^{t_N}\left(n_N \pm n_S\right)\right) + 4\left(3z^2+1\right)p_F^2\right)}{2n_Nn_S\left(8p_F^2\left(\mp 2n_Nn_S + 3z^2n_S^2 + 3z^2n_N^2\right) - 32zp_F^3(-1)^{t_N}\left(n_N \pm n_S\right) + 8zp_F(-1)^{t_N}\left(n_N \pm n_S\right)\left(3n_N^2 \mp 2n_Nn_S + 3n_S^2\right) - 48z^2p_F^4 + \left(n_N \pm n_S\right)^2\left(\mp 2\left(z^2+2\right)n_Nn_S + \left(z^2+4\right)n_S^2 + \left(z^2+4\right)n_N^2\right)\right)}$$

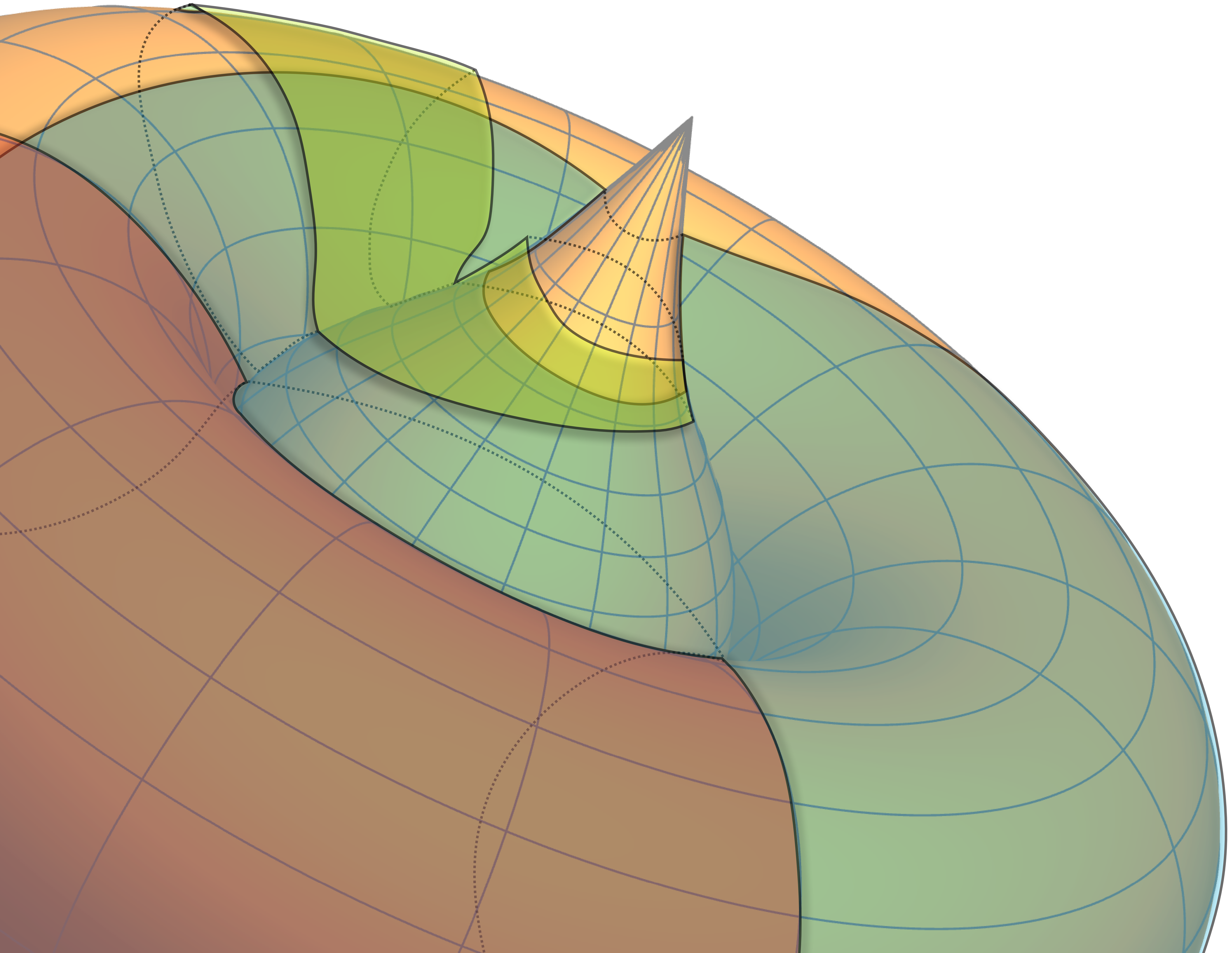
Two-dimensional Central Charge

N.B. $\mathbf{z} = \dfrac{p-q}{p+q}$

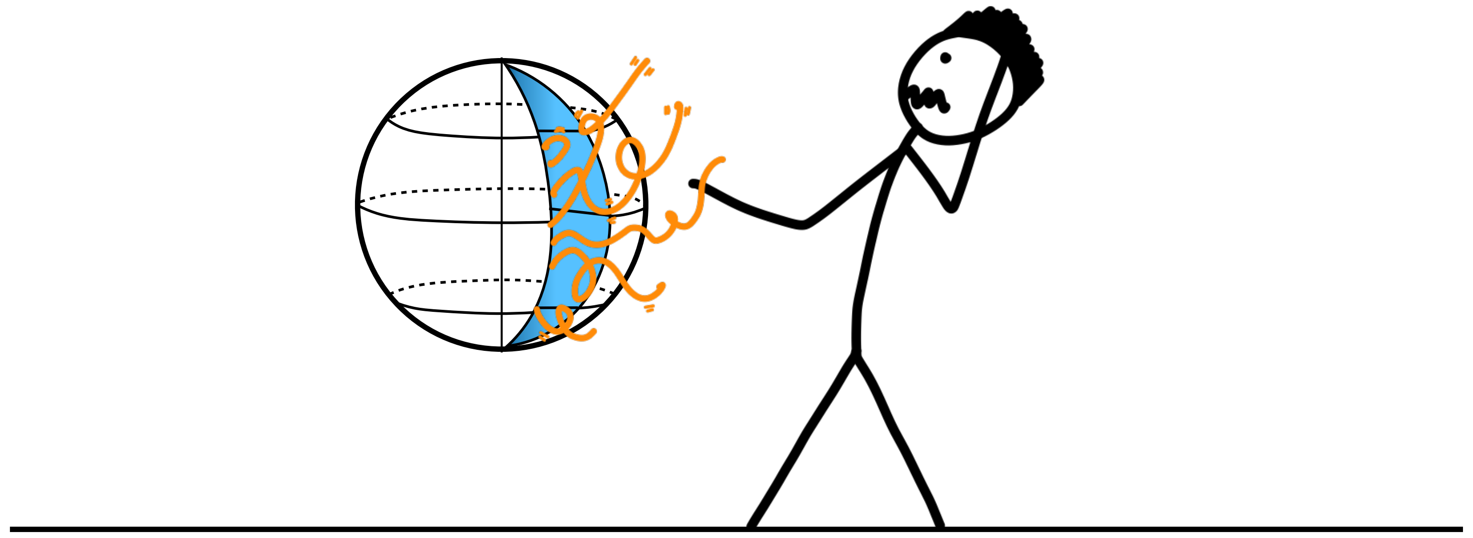
Twist ($+$) and anti-twist ($-$)

$$c_{2d}^{\pm} = \frac{N^3(g-1)\left(4p_F^2 - \left(n_N \pm n_S\right)^2\right)\left(2zp_F + (-1)^{t_N}\left(n_N \pm n_S\right)\right)\left((-1)^{t_N}\left(n_N \pm n_S\right)\left(16zp_F + \left(z^2+3\right)(-1)^{t_N}\left(n_N \pm n_S\right)\right) + 4\left(3z^2+1\right)p_F^2\right)}{2n_N n_S\left(8p_F^2\left(\mp 2n_N n_S + 3z^2 n_S^2 + 3z^2 n_N^2\right) - 32zp_F^3(-1)^{t_N}\left(n_N \pm n_S\right) + 8zp_F(-1)^{t_N}\left(n_N \pm n_S\right)\left(3n_N^2 \mp 2n_N n_S + 3n_S^2\right) - 48z^2 p_F^4 + \left(n_N \pm n_S\right)^2\left(\mp 2\left(z^2+2\right)n_N n_S + \left(z^2+4\right)n_S^2 + \left(z^2+4\right)n_N^2\right)\right)}$$

Checks with limiting cases $n_S = n_N = 1,\ p_F = 0$ of [Benini, Bobev (2013)]. In this limit $\mathbb{WCP}^1 \to \mathbb{P}^1$ and isometry enhances $\mathrm{U}(1) \to \mathrm{SU}(2)$ therefore $\epsilon \to 0$, i.e. no mixing in extremization.



The Bulk



Outline

General introduction

The boundary

The bulk

- $5d\ \mathcal{N} = 2$ gauged supergravity
- Consistent AdS_5 truncation with hypermultiplets
- Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N, n_S]}$
- Central charge from the poles and matching with field theory
- Numerical solutions

$5d\ \mathcal{N}=2$ gauged supergravity

$\mathcal{N}=2$ sugra admit coupling to n_V **vectors** and n_H **hypermultiplets**

Gravity and matter multiplets:

- **Gravity multiplet:** $\{e_\mu^a, \psi_\mu^i, A_\mu\}$ graviton, 2 gravitini, graviphoton
- **Vector multiplet:** $\{A_\mu^x, \lambda^{x\,i}, \phi^x\}$ vector field, 2 gauginos, real scalar
- **Hypermultiplet:** $\{q^X, \zeta^A\}$ 4 real scalars, 2 hyperinos

Total number of vector fields is $n_V+1\ A_\mu^I$

$5d\ \mathcal{N}=2$ gauged supergravity

Scalars in multiplets parametrize a moduli space $\mathscr{M} = \mathscr{S} \otimes \mathscr{Q}$

- Scalars in vector parametrize $\mathscr{S}$ a **very special real manifold** defined by cubic equation

$$\{h^I(\phi^x)\,|\,c_{IJK}h^Ih^Jh^K = 1\} \in \mathbb{R}^{n_V+1}$$

- Scalars in hyper parametrize $\mathscr{Q}$ a **quaternionic Kähler manifold** of real dimension $4n_H$

$5d\ \mathcal{N} = 2$ gauged supergravity

Gauging of **abelian isometries** of the quaternionic Kähaler by vectors A^I_μ generated by **Killing vectors** $k^X_I(q)$ that encode charge of scalars

$$D_\mu q^X = \partial_\mu q^X + g A^I_\mu k^X_I \qquad \text{where} \qquad k^X_I R^r_{XY} = D_Y P^r_I$$

Here we can define scalar super potential W to restore susy from prepotentials P^r_I.

Superpotential W relevant quantity for extremization! [Tachikawa (2006)].
Condition for $a-$maximization is condition on existence of AdS$_5$ solution.

$5d\ \mathcal{N} = 2$ gauged supergravity

Susy vacuum: $\partial_x W = 0, \partial_X W = 0$ from susy variations

$$\delta\psi_\mu = \left(F_\mu + \cdots + \frac{1}{2} g W \gamma_\mu \right) \epsilon = 0$$

$$\delta\lambda^x = \left(-\frac{i}{2}\gamma^\mu \partial_\mu \phi^x + \cdots + i\sqrt{\frac{3}{2}} g g^{xy} \partial_y W \right) \epsilon = 0$$

$$\delta\zeta^A = 0 \implies \left(-i\gamma^\mu \partial_\mu q^X + \cdots + \frac{3}{8} i g \partial_X W \right) \epsilon = 0$$

AdS_5 Truncation with Hypers

Starting point: consistent $5d$ truncation from $D = 11$

[Cassani, Josse, Petrini, Waldram (2010)]

- One hypermultiplet
- Two vector multiples

Broken global symmetries in FT related to massive vectors in sugra. Hypers serve as **Stuckelberg fields**. Massless vectors become massive by Higgs mechanism

AdS$_5$ Truncation with Hypers

11-dimensional metric ansatz

$$\mathrm{d}s_{11}^2 = e^{2\Delta}\mathrm{d}s_{\mathrm{AdS}_5}^2 + \mathrm{d}s_6^2, \quad \mathrm{d}s_6^2 = \bar{\Delta}^{1/2}e^{2g_0}\,\mathrm{d}s_{C_g}^2 + \frac{1}{4}\bar{\Delta}^{-2/3}\,\mathrm{d}s_4^2$$

$\mathscr{M}_6$ is fibration of squashed S^4 over Riemann surf C_g $\quad \mathscr{M}_4 \hookrightarrow \mathscr{M}_6 \to C_g$

Relation between warp factors $e^{2\Delta}R_{\mathrm{AdS}_5}^2 = e^{2f_0}\bar{\Delta}^{1/3}$

$\bar{\Delta}, f_0, g_0$ depend on $\mathbf{z}$ and curvature $\mathbf{k}$ of Riemann surface

AdS$_5$ Truncation with Hypers

Three vectors A_μ^I, $I = 0,1,2$. Scalar geometry parametrized by manifold

$$\mathbb{R}_+ \times SO(1,1) \times \frac{SU(2,1)}{SU(2) \times U(1)}$$

$$\{\Sigma, \phi\} \qquad \{\varphi, \Xi, \theta_1, \theta_2\}$$

AdS$_5$ Truncation with Hypers

Three vectors A_μ^I, $I = 0,1,2$. Scalar geometry parametrized by manifold

$$\{\Sigma, \phi\} \longleftarrow \mathbb{R}_+ \times \mathrm{SO}(1,1) \times \frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2) \times \mathrm{U}(1)} \longrightarrow \{\varphi, \Xi, \theta_1, \theta_2\}$$

AdS_5 Truncation with Hypers

Three vectors A_μ^I, $I = 0,1,2$. Scalar geometry parametrized by manifold

$$\mathbb{R}_+ \times SO(1,1) \times \frac{SU(2,1)}{SU(2) \times U(1)}$$

$$\{\Sigma, \phi\} \qquad \{\varphi, \Xi, \theta_1, \theta_2\}$$

Further truncation consistent with AdS_5 vacuum $\theta_1 = \theta_2 = 0$

Superpotential $W = \dfrac{\Sigma^3((\mathbf{k}e^{2\varphi} + 4)\cosh\phi - \mathbf{z}\mathbf{k}e^{2\varphi}\sinh\phi) + e^{2\varphi}}{4\Sigma^2}$

Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N,n_S]}$

Ansatz $\mathrm{d}s^2 = e^{2V(y)}\mathrm{d}s^2_{\mathrm{AdS}_3} + f(y)^2\mathrm{d}y^2 + h(y)^2\mathrm{d}z^2$ where (y, z) are coordinates on Spindle: $z \sim z + 2\pi$ and $y \in [y_N, y_S]$

Gauge fields: $A^{(I)} = a^{(I)}(y)\mathrm{d}z$ where $I = 1{,}2$

Field dependence on Spindle coordinates: $\Sigma(y), \phi(y), \varphi(y)$ and $\Xi = \bar{\Xi} \cdot z$

Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N, n_S]}$

Orthonormal frame of reference [Arav, Gauntlett, Roberts, Rosen (2022)]

$$e^a = e^V \bar{e}^a, \qquad e^3 = f\,\mathrm{d}y, \qquad e^4 = h\,\mathrm{d}z$$

Field strength becomes

$$f\,h\,F^{(I)}_{34} = \partial_y a^{(I)}$$

Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N, n_S]}$

Constants

Then, Maxwell's equations are

$$\frac{2e^{3V}}{3\Sigma^2}\left[(\cosh 2\phi - \mathbf{z}\sinh 2\phi)F_{34}^{(1)} + (\mathbf{z}\cosh 2\phi - \sinh 2\phi)F_{34}^{(2)}\right] = \mathscr{E}_1$$

$$\frac{2e^{3V}}{3\Sigma^2}\left[\mathbf{zk}\Sigma^6 F_{34}^{(0)} - (\cosh 2\phi + \mathbf{z}\sinh 2\phi)F_{34}^{(1)} + (\mathbf{z}\cosh 2\phi + \sinh 2\phi)F_{34}^{(2)}\right] = \mathscr{E}_2$$

$$\partial_y\left(\frac{1}{3}e^{3V}\Sigma^4 F_{34}^{(0)}\right) = \frac{1}{4}e^{4\psi+3V} g\, f\, h^{-1} D_z\Xi \qquad D_z\Xi = \bar{\Xi} + \underbrace{g(a^{(0)} + \mathbf{zk}a^{(1)} - \mathbf{z}a^{(2)})}_{\textbf{Higgsed combination}}$$

Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N,n_S]}$

BPS equations for this geometry computed by factorizing Killing spinors

$$\epsilon = \psi \otimes \chi : \quad \begin{cases} \nabla_m \psi = -\frac{\kappa}{2}\Gamma_m \psi, \;\; \kappa = \pm 1 \\[2ex] \chi = e^{\frac{V}{2}} e^{isz} \begin{pmatrix} \sin\frac{\xi}{2} \\[1ex] \cos\frac{\xi}{2} \end{pmatrix} \end{cases}$$

Chirality of dual SCFT
$\mathcal{N} = (0,2)$ or $\mathcal{N} = (2,0)$

Where ψ spinor on Spindle and χ spinor on AdS_3

Down to $\mathrm{AdS}_3 \times \mathbb{WCP}^1_{[n_N, n_S]}$

BPS equations

$$\xi' = 2f(gW\cos\xi + \kappa e^{-V}), \qquad \phi' = -2fg\sin\xi\,\partial_\phi W$$

$$3V' = 2fgW\sin\xi, \qquad \varphi' = -fg\sin^{-1}\xi\partial_\varphi W$$

$$3\Sigma' = -2fg\,\Sigma^2\sin\xi\,\partial_\Sigma W, \qquad 3h' = 2fh\sin^{-1}\xi(gW(1+2\cos^2\xi) - 3\kappa e^{-V}\cot\xi)$$

Algebraic constraints

$$\sin\xi(s - Q_z) = -h(gW\cos\xi + \kappa e^{-V}) \qquad D_\mu\epsilon = (\nabla_\mu - iQ_\mu)\epsilon$$

$$gh\partial_\varphi W\cos\xi = \partial_\varphi Q_z\sin\xi$$

$\mathbb{Z}_2$ symmetry

$$\{h \to -h, a^{(I)} \to -a^{(I)}, Q_z \to -Q_z, s \to -s, \phi \to -\phi, \mathbf{z} \to -\mathbf{z}\}$$

Central Charge from the Poles

BPS equations give $h = k e^V \sin\xi$, k constant

Conditions at poles are enough [Amariti, Petri, Segati (2023)] [Suh (2023)] …

1. $\cos\xi\,|_{N,S} = (-1)^{t_{N,S}}$ where $t_{N,S} \in \{0,1\}$ twist or anti-twist
2. $k \sin'\xi\,|_{N,S} = \dfrac{(-1)^{l_{N,S}}}{n_{N,S}}$ where $l_N = 0$, $l_S = 1$ due to $\mathbb{Z}_2$ symmetry
3. $(s - Q_z)\,|_{N,S} = \dfrac{1}{2n_{N,S}}(-1)^{t_{N,S}+l_{N,S}+1}$ from BPS equations
4. $\partial_\varphi W = 0$ to ensure finiteness of $\varphi(y)$

Central Charge from the Poles

Fluxes can be written int terms of pole data

$$\frac{p_I}{n_N n_S} = \frac{1}{2\pi}\int_{\mathbb{WCP}} gF^{(I)} = g\mathcal{I}^{(I)}|_N^S \qquad \mathcal{I}^{(I)} \equiv -ke^V \cos\xi h^I$$

Flavour flux $\quad p_F = p_1 = g n_N n_S \mathcal{I}^{(1)}|_N^S$

R-symmetry flux $\quad p_R = -p_2 = \frac{1}{2}(n_S(-1)^{t_N} + n_N(-1)^{t_S})$

Constraint $\quad p_M \propto p_0 + \mathbf{zk}p_1 - \mathbf{k}p_2 = 0$

Central Charge from the Poles

Equations before fix boundary conditions for V, h, ϕ, Σ. Moreover

$$e^{V(y)} f(y) h(y) = -\frac{k}{2\kappa}\left(e^{3V(y)}\cos\xi(y)\right)'$$

Very important!!

Central charge

$$c_{2d} = \frac{3}{2G_5}\Delta z \int_{y_n}^{y_s} e^{V(y)}\,|\,f(y)h(y)\,|\,\mathrm{d}y$$

Central Charge from the Poles

Central charges match with the FT ones! Both for twist and anti-twist

Central Charge from the Poles

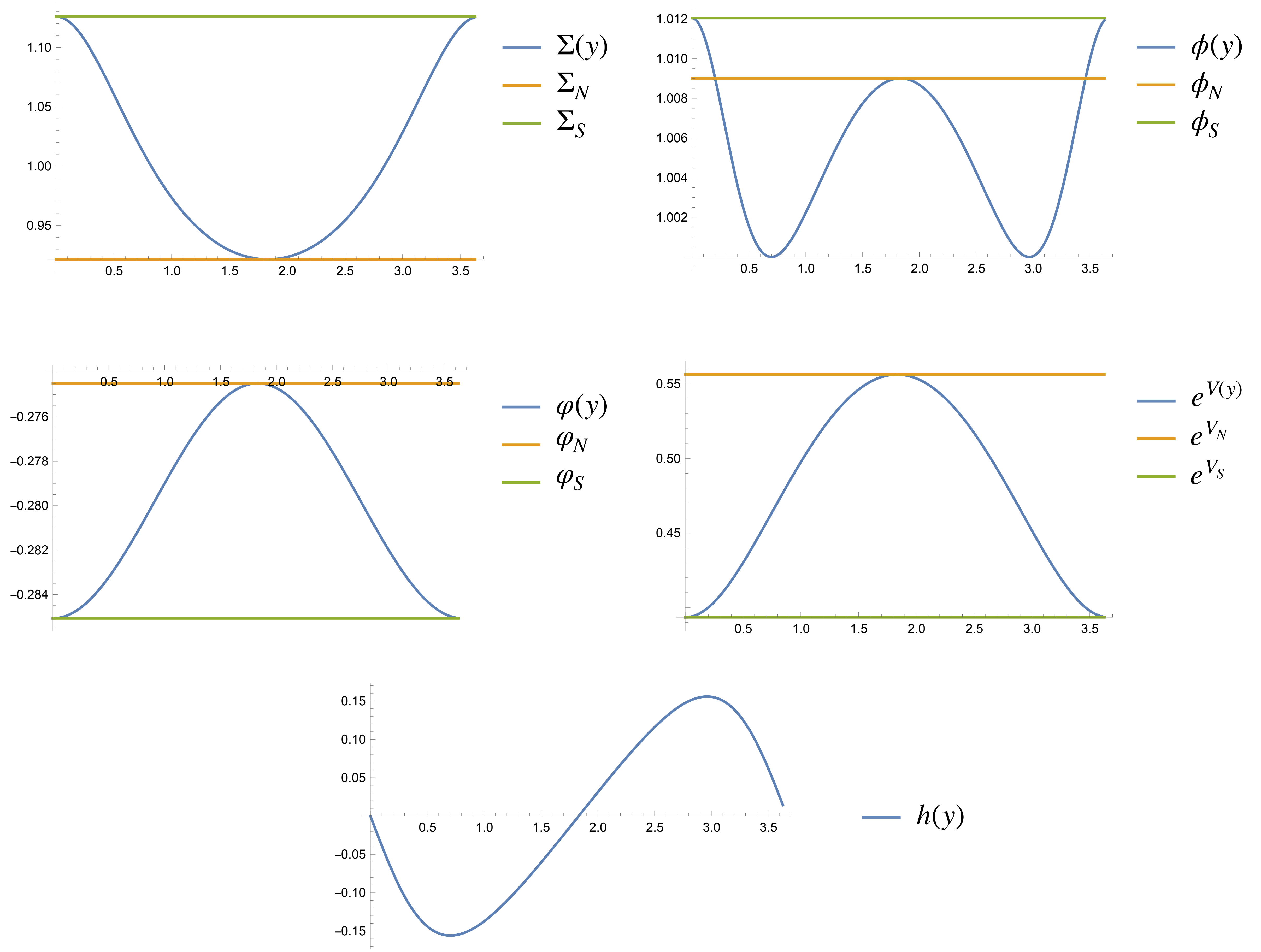
Central charges match with the FT ones! Both for twist and anti-twist

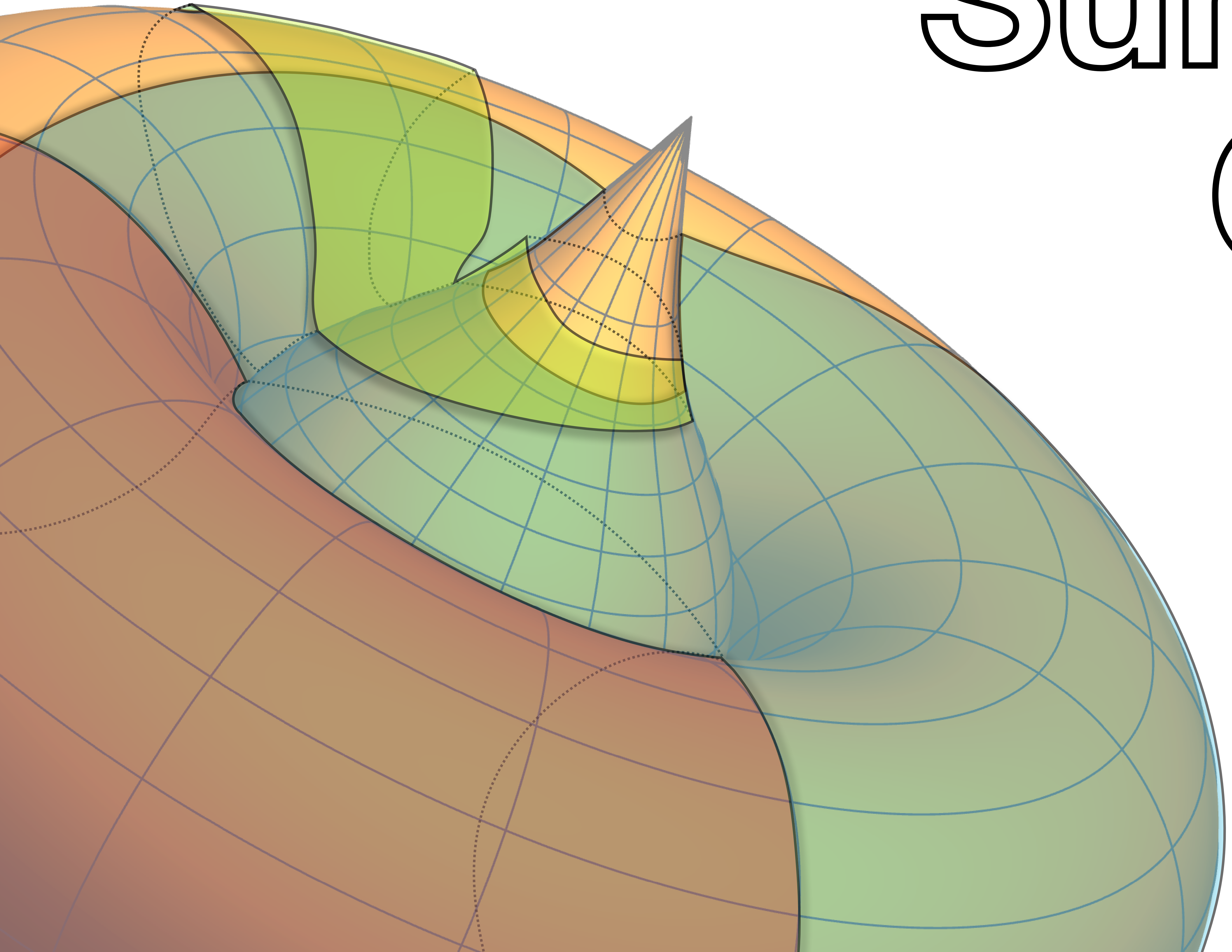
We found analytic solution by restricting to **graviton sector** only for anti-twist case with $\mathbf{k} = -1$ and generic z matching [Ferrero, Gauntlett, Ipina, Martelli, Sparks (2020)] [Ferrero, Gauntlett, Sparks (2021)]. Graviton sector fixes p_F.

Numerical solutions

For generic p_F (consistent with quantization) we find numerical solution by integrating BPS eqns. [Arav, Gauntlett, Roberts, Rosen (2022)] [Amariti, Petri, Segati (2023)] [Suh (2023)]

Still only solutions for $\mathbf{z} = -1$ and anti-twist





Summary and Outlook

Summary

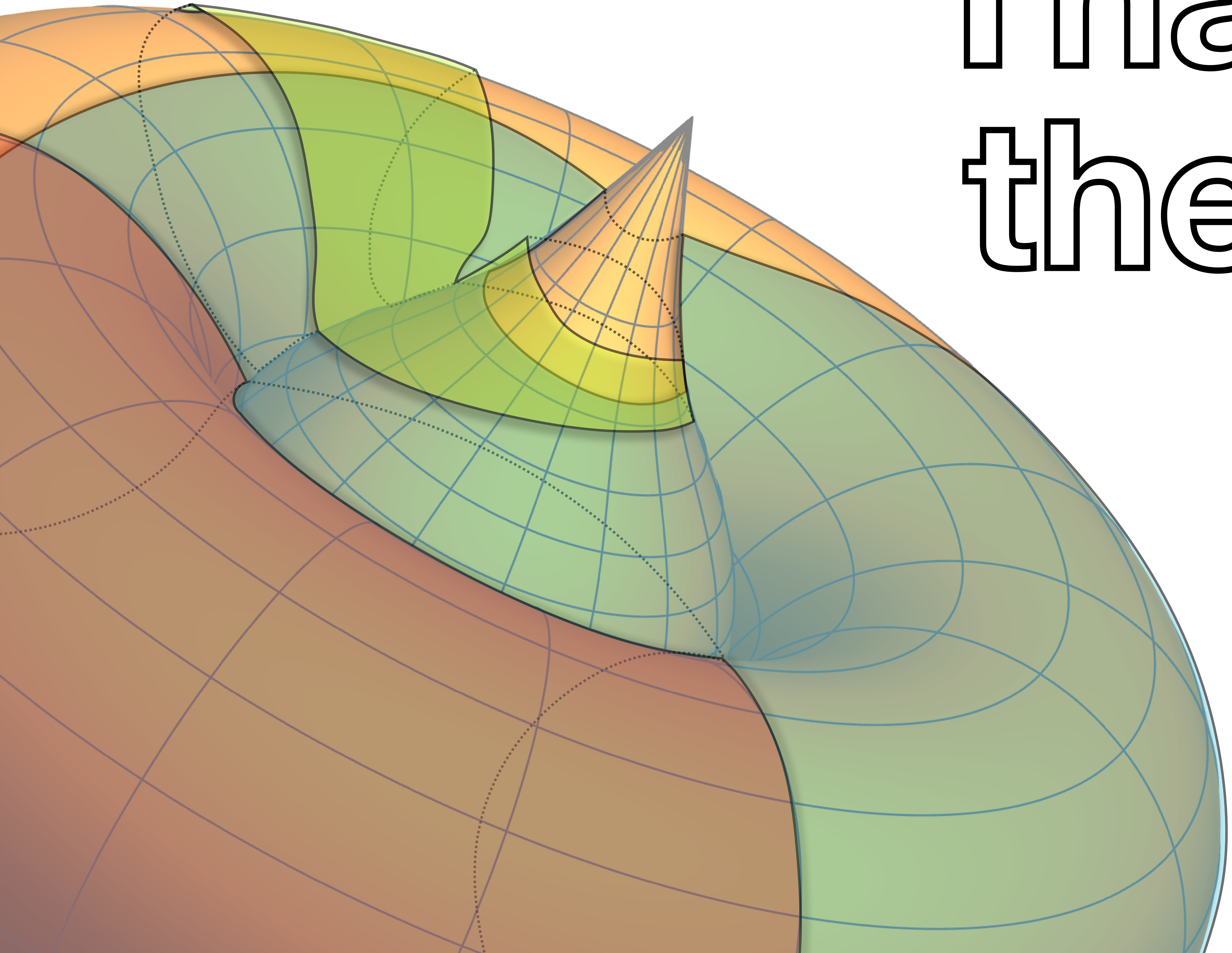
We provided a precision test for the AdS/CFT correspondence by

- Computing the central charge of the $2d$ field theory by reduction
- Analizing the $\mathrm{AdS}_3 \times \mathbb{WCP}^1$ susy solution to $5d$ gauged supergravity in presence of hypermultiplets
- The central charge can be extracted solely from the contribution on the poles of the spindle
- Matching the (very intricate) central charge between the gravity and field theory side

Outlook

Some future avenues

- Computing the sub-leading order contributions to the central charge. Doable in field theory, very complicated in gravity
- AdS_4 truncations with hypers and their compactifications on spindles
- Compute this model from $11d$ by means of equivariant localization. Similarly to $AdS_3 \times M_8$ solutions of [Benetti Genolini, Gauntlett, Sparks (2023)]



Thank You for the Attention