BBBW ON THE SPINDLE

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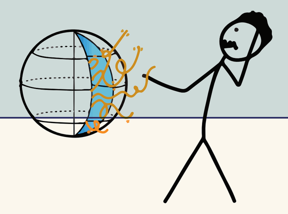
ABSTRACT

We study the spindle compactification of families of AdS_5 consistent truncations corresponding to M5-branes wrapped on complex curves in Calabi-Yau three-folds.
From the AdS/CFT correspondence these models are dual to N = 1 SCFTs obtained by gluing of T_N blocks. The truncations considered here have both vector and hyper multiplets and the analysis of the BPS equations on the spindle allows to extract the central charges. Such analysis gives also consistency conditions for the existence of the solutions.
The solutions are then found both analytically and numerically for opportune choices of the charges for some sub-families of truncations. We then compare our results with the one expected from the field theory side, by integrating the anomaly polynomial.

BULK



CONSISTENT AdS_5 TRUNCATION WITH HYPERS

Starting point: consistent 5d trunctation from 11d [6] with ***one hyper and two vectors***. Ansatz geometry is warped product of AdS_5 and squashed S^4 fibered over the C_g.

$$M_4 \hookrightarrow M_6 \xrightarrow{\pi} C_g$$

Vector	Σ, ϕ	$\mathcal{M}_V = \mathbb{R}_+ \times \mathrm{SO}(1,1)$
Hypers	$\varphi, \Xi, \theta_1, \theta_2$	$\mathcal{M}_H = \dfrac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times \mathrm{U}(1)}$

$$W = \frac{\Sigma^3((\mathbf{k}e^{2\varphi}+4)\cosh\phi - \mathbf{zk}e^{2\varphi}\sinh\phi) + e^{2\varphi}}{4\Sigma^2}$$

Further truncation setting $\theta_1 = \theta_2 = 0$ — Superpotential

DOWN TO AdS_3 X WCP^1

Ansatz for the geometry $\mathrm{d}s^2 = e^{2V(y)}\mathrm{d}s^2_{\mathrm{AdS}_3} + f(y)^2\mathrm{d}y^2 + h(y)^2\mathrm{d}z^2$ with $z \sim z + 2\pi,\ y \in [y_N, y_S]$

Ansatz for the fields $\underset{I=0,1,2}{A^{(I)}} = a^{(I)}(y)\mathrm{d}z \quad \Sigma(y),\ \phi(y),\ \varphi(y),\ \Xi = \bar{\Xi}\cdot z$

Maxwell's equations

$$\frac{2e^{3V}}{3\Sigma^2}\left[(\cosh 2\phi - \mathbf{z}\sinh 2\phi)F^{(1)}_{34} + (\mathbf{z}\cosh 2\phi - \sinh 2\phi)F^{(2)}_{34}\right] = \mathcal{E}_1$$

$$\frac{2e^{3V}}{3\Sigma^2}\left[\mathbf{zk}\Sigma^6 F^{(0)}_{34} - (\cosh 2\phi + \mathbf{z}\sinh 2\phi)F^{(1)}_{34} + (\mathbf{z}\cosh 2\phi + \sinh 2\phi)F^{(2)}_{34}\right] = \mathcal{E}_2$$

$$\partial_y\left(\frac{1}{3}e^{3V}\Sigma^4 F^{(0)}_{34}\right) = \frac{1}{4}e^{4\psi+3V} g\, f\, h^{-1} D_z\Xi$$

BPS equations

$$\xi' - 2f(gW\cos\xi + \kappa e^{-V}) = 0$$
$$V' - \frac{2}{3}fgW\sin\xi = 0$$
$$\Sigma' + \frac{2}{3}fg\,\Sigma^2\sin\xi\,\partial_\Sigma W = 0$$
$$\phi' + 2fg\sin\xi\,\partial_\phi W = 0$$
$$\varphi' + \frac{fg}{\sin\xi}\partial_\varphi W = 0$$
$$h' - \frac{2fh}{3\sin\xi}(gW(1+2\cos^2\xi) + 3\kappa e^{-V}\cot\xi) = 0$$

CENTRAL CHARGES FROM THE POLES

Central charge obtained from pole analysis, without solving BPS equations [7-9]

$\partial_\varphi W|_{N,S} = 0$ To ensure that φ stays finite. "Attractor mechanism" as maximisation

Magnetic fluxes can be expressed in terms of the pole data $F^{(I)}_{yz} = (a^{(I)})'$

$$\frac{p_I}{n_N n_S} = \frac{1}{2\pi}\int_{\mathbb{WCP}^1} gF^{(I)} = ga^{(I)}|^S_N$$

$$p_R = \frac{1}{2}(p_0+p_1+p_2) = \frac{1}{2}\left(n_S(-1)^{t_N} + n_N(-1)^{t_S}\right)$$
$$p_F = \frac{3}{4}(p_2 - p_1)$$
$$p_M \propto 2p_0 - p_1 - p_2 = 0$$

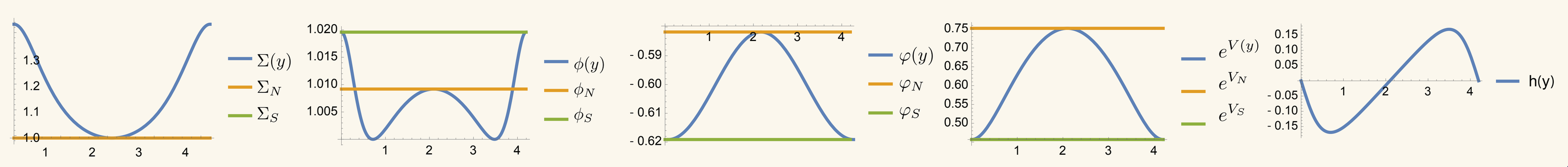
Last constraint is Higgsing of vector multiplet associated with the ***broken global symmetry*** in dual field theory

BROWN-HENNEAUX AND NUMERICS

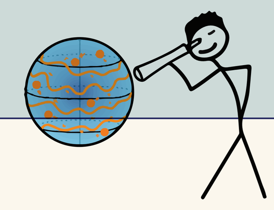
By conditions at poles find central charge with Brown-Hanneaux formula

$$e^{V(y)}f(y)h(y) = -\frac{k}{2\kappa}\left(e^{3V(y)}\cos\xi(y)\right)' \quad \text{Total derivative!} \quad c_{2d} = \frac{3}{2G_5}\Delta z\int_{y_n}^{y_s} e^{V(y)}|f(y)h(y)|\,\mathrm{d}y$$

Perfect match with field theory. If analytic solution to BPS eqns exists, then the central charge of the theory is the one found here. Analitic solution in the pure gravity sector. Checks with numerical solutions of BPS eqns for generic flavour flux. Solutions only for z=-1 and anti-twist

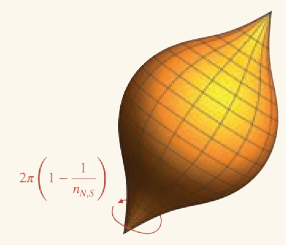


BOUNDARY



THE SPINDLE AND COMPACTIFICATIONS

A Spindle is an example of a supersymmetry preserving ***orbifold***. Locally is a Z_2-orbifold of an S^2 with conical singularities at the poles. Spindles provided interesting geometry for compactifications, where ***supersymmetry is preserved in a "non-trivial" way***.
Beside the Maldacena-Nunez topological twist [1-3], one can turn on a different flux for the R-symmetry background dubbed ***anti-twist***.



$$\frac{1}{2\pi}\int_{\mathbb{WCP}^1_{[n_N,n_S]}} F^R = \frac{n_N+n_S}{n_N n_S} = \chi\left(\mathbb{WCP}^1_{[n_N,n_S]}\right)$$

Twist

$$\frac{1}{2\pi}\int_{\mathbb{WCP}^1_{[n_N,n_S]}} F^R = \frac{n_N-n_S}{n_N n_S}$$

Anti-Twist

THE M5-BRANE ANOMALY POLYNOMIAL

The world-volume theory of an M5-brane is a ***6d $\mathcal{N}$ = (2,0) theory***. The anomaly comes from the ***four components chiral spinors*** and a ***self-dual two form*** [5].

$$\mathcal{I}_D = \frac{1}{2}\mathrm{ch}\,S(N)\hat{A}(TW)$$
$$\mathcal{I}_A = \frac{1}{5760}\left(16p_1(TW)^2 - 112p_2(TW)\right)$$

Spinors are sections of a rank-4 spinor bundle constructed from the normal bundle N using the spinor representation of SO(5). Â(TW) is the ***Dirac genus*** of the tangent space to the world-volume, i.e. the index of the Dirac operator.

A stack of M5-branes contributes with the following anomaly. Last pice from CS in 11d

$$\mathcal{I}_8 = \frac{N-1}{48}\left[p_2(NW) - p_2(TW) + \frac{1}{4}(p_1(TW) - p_1(NW))^2\right] + \frac{N^3-N}{24}p_2(NW)$$

THE 4D GEOMETRY

The starting point is a family of 4d theories arising from ***wrapping M5-branes on Riemann surfaces in Calabi-Yau three-folds*** [4]. When the Calabi-Yau is decomposable, the discrete chioices of the degrees of the line bundles in the local geometry label different IR dynamics of the effective 4d theories. By ***integrating the M5-brane anomaly on the Riemann surface*** one finds 4d anomalies labelled by the degrees.

$$\mathbb{C}^2 \hookrightarrow \mathcal{L}_1 \oplus \mathcal{L}_2 \xrightarrow{\pi} C_g$$

$$c_1(\mathcal{L}_1) = p \qquad c_2(\mathcal{L}_2) = q \qquad p + q = 2g - 2$$

$$\mathcal{I}_6 = \int_{C_g}\mathcal{I}_8 \sim \sum_{i,j,k=1,2} A_{ijk}c_1(F_i)c_1(F_j)c_1(F_k)$$

Expansion coefficients are the ***'t Hooft anomalies*** between two abelian symmetries.

DOWN TO TWO-DIMENSIONS

The last step is to integrate the anomaly of the 4d effective theories, at ***large-N***, on the Spindle. We need to turn on ***fluxes*** for the flavor symmetry and the R-symmetry. The latter will depend on the choice of twist or anti-twist background. Moreover there the spindle carries an ***additional abelian symmetry*** coming from the rotation isometry around the azimuthal axis. The trial 2d central charge can be found by mixing the three abelian symmetries. The exact central charge is then found by ***c-extremization***.

$$\int c_1(F_R) = \frac{p_R}{n_N n_S} = \frac{(-1)^{t_N}}{n_N} - \frac{(-1)^{t_S+1}}{n_S}, \qquad \int c_1(F_F) = \frac{p_F}{n_N n_S}$$

Where t_N = 0,1 and $t_S = t_N$ for the twist or $t_S = t_N+1$ for the anti-twist.

THE 2D CENTRAL CHARGE: FIELD THEORY AND SUPERGRAVITY

$$c^T_{2d} = \frac{(g-1)N^3\left(4p_F^2 - (n_N+n_S)^2\right)\left(2zp_F + (-1)^{t_N}(n_N+n_S)\right)\left((-1)^{t_N}(n_N+n_S)\left(16zp_F + (z^2+3)(-1)^{t_N}(n_N+n_S)\right) + 4\left(3z^2+1\right)p_F^2\right)}{2n_N n_S\left(8p_F^2\left(-2n_N n_S + 3z^2 n_S^2 + 3z^2 n_N^2\right) - 32zp_F^3(-1)^{t_N}(n_N+n_S) + 8zp_F(-1)^{t_N}(n_N+n_S)\left(3n_N^2 - 2n_N n_S + 3n_S^2\right) - 48z^2p_F^4 + (n_N+n_S)^2\left(-2\left(z^2+2\right)n_N n_S + \left(z^2+4\right)n_S^2 + \left(z^2+4\right)n_N^2\right)\right)}$$

$$c^{AT}_{2d} = \frac{(g-1)N^3\left((n_S-n_N)^2 - 4p_F^2\right)\left(2zp_F + (-1)^{t_N}(n_N-n_S)\right)\left((-1)^{t_N}(n_N-n_S)\left(16zp_F + (z^2+3)(-1)^{t_N}(n_N-n_S)\right) + 4\left(3z^2+1\right)p_F^2\right)}{2n_N n_S\left(8p_F^2\left(2n_N n_S + 3z^2 n_S^2 + 3z^2 n_N^2\right) + 32zp_F^3(-1)^{t_N}(n_S-n_N) - 8zp_F(-1)^{t_N}(n_S-n_N)\left(3n_N^2 + 2n_N n_S + 3n_S^2\right) - 48z^2p_F^4 + (n_S-n_N)^2\left(2\left(z^2+2\right)n_N n_S + \left(z^2+4\right)n_S^2 + \left(z^2+4\right)n_N^2\right)\right)}$$

REFERENCES

[1] P. Ferrero, J. P. Gauntlett, J. M. Pérez Ipiña, D. Martelli and J. Sparks, *D3-Branes Wrapped on a Spindle*, Phys. Rev. Lett. 126 (2021) 111601 [2011.10579]
[2] P. Ferrero, J. P. Gauntlett, J. M. P. Ipiña, D. Martelli and J. Sparks, *Accelerating black holes and spinning spindles*, Phys. Rev. D 104 (2021) 046007 [2012.08530]
[3] P. Ferrero, J. P. Gauntlett and J. Sparks, *Supersymmetric spindles*, JHEP 01 (2022) 102 [2112.01543]
[4] I. Bah, C. Beem, N. Bobev and B. Wecht, *Four-Dimensional SCFTs from M5-Branes*, JHEP 06 (2012) 005 [1203.0303]
[5] E. Witten, *Five-brane effective action in M-theory*, J. Geom. Phys. 22, 103-133 (1997) [hep-th/9610234]
[6] D. Cassani, G. Josse, M. Petrini and D. Waldram, *$\mathcal{N}$ = 2 consistent truncations from wrapped M5-branes*, JHEP 02 (2021) 232 [2011.04775]
[7] M. Suh, *Baryonic spindles from conifolds*, 2304.03308
[8] A. Amariti, N. Petri and A. Segati, *$T^{1,1}$ truncation on the spindle*, JHEP 07 (2023) 087 [2304.03663]
[9] I. Arav, J. P. Gauntlett, M. M. Roberts and C. Rosen, *Leigh-Strassler compactified on a spindle*, JHEP 10 (2022) 067 [2207.06427]