LEZIONE 3 : STUDI DI FUNZIONE

Schema:
- Dominio + parità
- intersezioni con gli assi
- segno
- limiti $\longrightarrow$ "bordi del dominio"
- derivata prima $\rightarrow$ zeri, segno (max, min, pti stazionari) — monotonia
- derivata seconda $\rightarrow$ zeri, segno (flessi, concavità)

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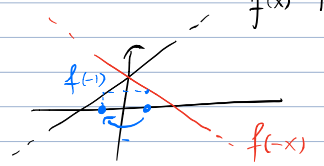
36) $f(x) = x^4 - 3x^2 + 4$

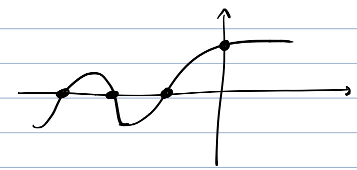
- DOMINIO $f = \mathbb{R}$ $\qquad D = \mathbb{R}$

PARITÀ $f(-x) = (-x)^4 - 3(-x)^2 + 4 = x^4 - 3x^2 + 4 = f(x)$

$f(x)$ $\quad f(-x) = f(x)$ PARI $\longrightarrow$ simm. sotto riflessioni rispetto asse verticale



- INTERSEZIONI ASSI



ASSE Y :
$$\begin{cases} x = 0 \\ f(x) = x^4 - 3x^2 + 4 \end{cases}$$

$$f(0) = 0 - 0 + 4 = 4$$

$$I_y = (0, 4)$$

ASSE X :
$$\begin{cases} y = 0 \\ y = f(x) = x^4 - 3x^2 + 4 \end{cases}$$

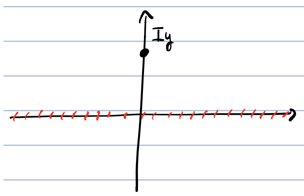
CAMBIO DI VARIABILE

$$x^4 - 3x^2 + 4 = 0 \longrightarrow x^2 = t$$

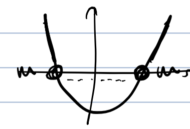
$$t^2 - 3t + 4 = 0 \longrightarrow t_{1,2} = \frac{3 \pm \sqrt{9 - 16}}{2} \notin \mathbb{R}$$

$\Rightarrow$ la nostra eq non ha soluzioni reali

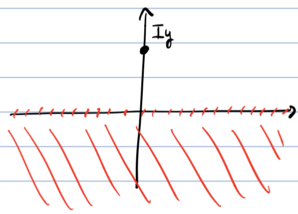
$\Rightarrow$ non ci sono intersezioni con l'asse x



• SEGNO : $f(x) > 0$ $\qquad x^4 - 3x^2 + 4 > 0$



$\downarrow$ eq. associata

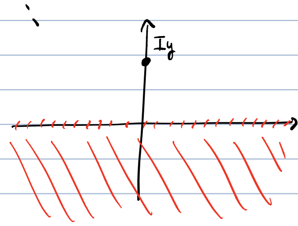


$x^4 - 3x^2 + 4 = 0$ non ha soluzioni e in più coeff del termine $t^2 = x^4$ è positivo

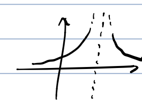
$\Rightarrow$ soluzione di $f(x) > 0$ è $\forall x \in \mathbb{R}$

• BORDI DEL DOMINIO $\quad D = \mathbb{R}$ se $x \to \pm\infty$

$$\lim_{x \to \pm\infty} \left(x^4 - 3x^2 + 4\right) = \lim_{x \to \pm\infty} x^4 \left(1 - \underbrace{\frac{3}{x^2}}_{\to 0} + \underbrace{\frac{4}{x^4}}_{\to 0}\right) = \lim x^4 = +\infty$$

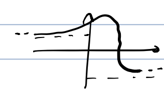


ASINTOTI : – VERTICALI



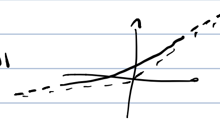
NON NE ABBIAMO

– ORIZZONTALI



NON NE ABBIAMO

– OBLIQUI



POTREMMO AVERNE

$\hookrightarrow$ VEDIAMO DOPO CON DERIVATA

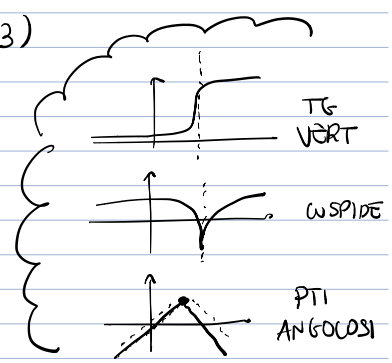
$f(x) = x^4 - 3x^2 + 4$

• DERIVATA PRIMA $\quad f'(x) = 4x^3 - 3 \cdot 2x + 0 = 4x^3 - 6x =$

$= 2x(2x^2 - 3)$

• dominio D'

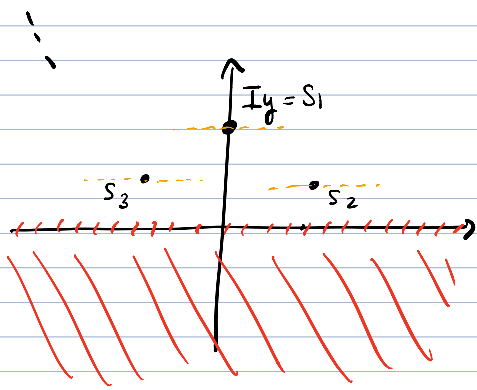
$D' = \mathbb{R}$



• zeri $f'(x) = 2x(2x^2-3) = 0$

o $x = 0$

oppure $2x^2-3=0 \longrightarrow x^2 = \frac{3}{2} \longrightarrow x = \pm\sqrt{\frac{3}{2}}$



pti stazionari

$S_1 = (0, f(0)) = (0,4) = I_y$

$S_2 = \left(\sqrt{\frac{3}{2}}, f\left(\sqrt{\frac{3}{2}}\right)\right) = \left(\sqrt{\frac{3}{2}}, \frac{7}{4}\right)$

$f\left(\sqrt{\frac{3}{2}}\right) = \frac{9}{4} - 3\cdot\frac{3}{2} + 4 =$

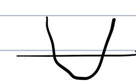
$= \frac{9-18+16}{4} = \frac{7}{4}$

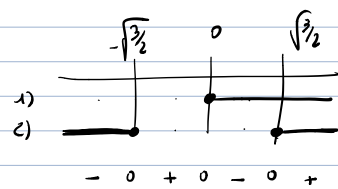
$S_3 = \left(-\sqrt{\frac{3}{2}}, f\left(-\frac{\sqrt{3}}{2}\right)\right) = \left(-\sqrt{\frac{3}{2}}, \frac{7}{4}\right)$

• segno $f'(x) = 2x(2x^2-3)$

1) $2x > 0$ se $x > 0$

2) $2x^2-3>0$ se $x < -\sqrt{\frac{3}{2}} \vee x > \sqrt{\frac{3}{2}}$



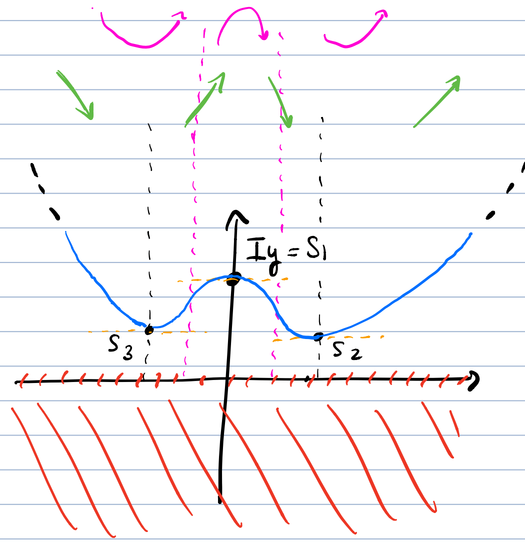


$f'(x) > 0$ se $-\sqrt{\frac{3}{2}} < x < 0 \ \vee \ x > \sqrt{\frac{3}{2}}$

COMPITO

• asintoti obliqui $\lim_{x\to\pm\infty} f'(x) = \lim_{x\to\pm\infty} (4x^3-6x) = \dots = \pm\infty$

$\Longrightarrow$ NON CI SONO ASINTOTI OBLIQUI

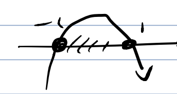


COMPITO STUDIO DI f'' e CONCAVITÀ

45] $f(x) = x\sqrt{1-x^2}$

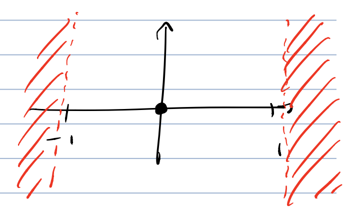
concavità negativa

- dominio : $1-x^2 \geq 0$



$1-x^2=0 \longrightarrow x^2=1 \longrightarrow x=\pm 1$

$\hookrightarrow D=[-1,1]$



- parità $f(-x) = -x\sqrt{1-(-x)^2} = -x\sqrt{1-x^2} = -f(x)$

DISPARI : siamo rispetto a riflessioni rispetto a O

$f(x) = x+5$
$f(-x) = -x+5 \neq \pm f(x)$

- intersezioni $\underline{Y|}$ $f(0) = 0\cdot\sqrt{1-0} = 0$ $I_y = O$

dispari $\Rightarrow$ $f(x) = -f(-x)$ $x=0$

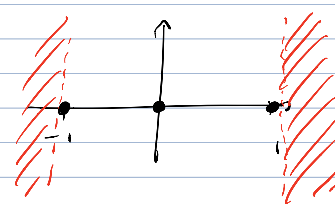
$f(0) = -f(0) \Longrightarrow f(0)=0$

$\left[\begin{array}{l} 1/x \text{ é dispari} \\ \text{ma non passa da} \\ 0 \end{array} \right.$

$$\cap x \begin{cases} x\sqrt{1-x^2} = 0 \\ -1 \leq x \leq 1 \end{cases}$$

$x = 0$
oppure
$\sqrt{1-x^2} = 0 \rightsquigarrow x = \pm 1$

$$I_x = 0, (1,0), (-1,0)$$



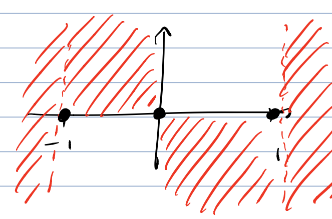
- segno $\begin{cases} f(x) > 0 \\ -1 \leq x \leq 1 \end{cases}$ $\quad x\sqrt{1-x^2} > 0$

$\downarrow$ dove $\sqrt{1-x^2} \neq 0$, se $x \neq \pm 1$

$x > 0$

$\Rightarrow f = 0$ in $x = 0, \pm 1$

$f > 0$ se $0 < x < 1$



- bordi del dominio $x = \pm 1 \rightsquigarrow (\pm 1, 0)$
- derivata prima $f'(x) = 1 \cdot \sqrt{1-x^2} + x \cdot [(1-x^2)^{1/2}]' =$ derivata funz. composta

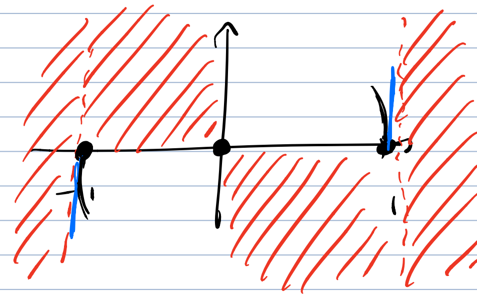
$$= \sqrt{1-x^2} + x \cdot \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) =$$

$$= \sqrt{1-x^2} - x^2 \frac{1}{\sqrt{1-x^2}} = \frac{1-x^2-x^2}{\sqrt{1-x^2}} =$$

$$= \frac{1-2x^2}{\sqrt{1-x^2}}$$

- dominio $D' = (-1,1) \rightsquigarrow$ estremi esclusi

$\hookrightarrow \lim_{x\to\pm1} f'(x) = \lim_{x\to 1^-} \frac{1-2x^2}{\sqrt{1-x^2}} \left(= \frac{-1}{0^+}\right) = -\infty$



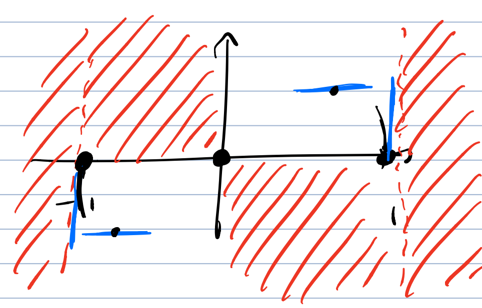
- pti stazionari $f'(x)=0$

$$\begin{cases} \dfrac{1-2x^2}{\sqrt{1-x^2}} = 0 \\ -1<x<1 \end{cases} \rightsquigarrow 1-2x^2=0 \quad x^2=\frac{1}{2}$$

$$x = \pm\frac{1}{\sqrt{2}}$$

$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$f\left(-\frac{1}{\sqrt{2}}\right) \underset{\text{dispari}}{=} -\frac{1}{2} \longrightarrow \text{PTI STAZ } \left(\frac{1}{\sqrt{2}},\frac{1}{2}\right) \text{ e } \left(-\frac{1}{\sqrt{2}},-\frac{1}{2}\right)$$



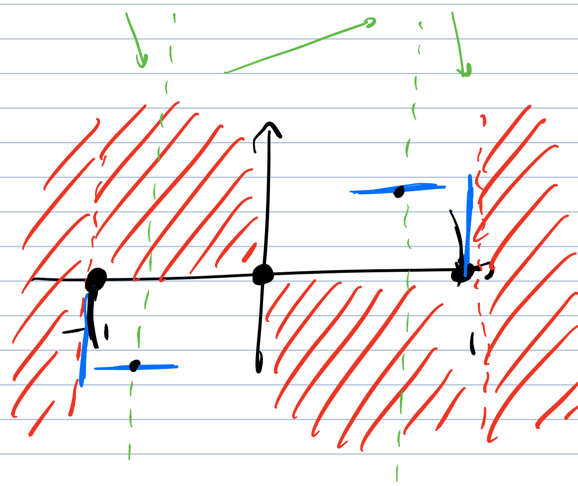
• segno $f'(x)$

$$f'(x) > 0 \qquad \frac{1-2x^2}{\sqrt{1-x^2}} > 0 \longrightarrow 1-2x^2 > 0$$

↑ positivo, $\neq 0$ se $\{-1<x<1\} = D'$

eq. associata $\quad \overset{\text{conc. neg.}}{1-2x^2} = 0 \longrightarrow x = \pm\frac{1}{\sqrt{2}}$

$f' > 0$ se $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$



• derivata seconda $\qquad f'(x) = \frac{1-2x^2}{\sqrt{1-x^2}}$

$$f''(x) = \frac{1}{1-x^2}\left[-4x\sqrt{1-x^2} + (1-2x^2)\cdot\frac{1}{\cancel{2}}\frac{1}{\sqrt{1-x^2}}\cdot(+\cancel{2}x)\right] =$$

$$= \frac{1}{1-x^2}\left[-4x\sqrt{1-x^2} + x\frac{1-2x^2}{\sqrt{1-x^2}}\right] =$$

$$= \frac{1}{(1-x^2)^{3/2}}\left(-4x(1-x^2) + (1-2x^2)x\right)$$

$$= \frac{1}{(1-x^2)^{3/2}} \left(-4x + 4x^3 + x - 2x^3 \right) =$$

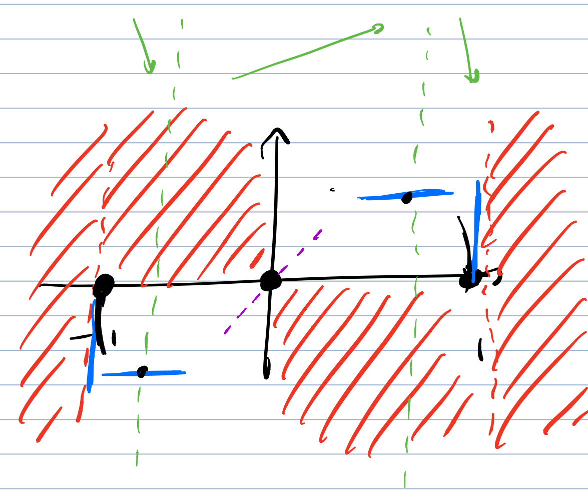
$$= \frac{(2x^2-3)x}{(1-x^2)^{3/2}}$$

- dominio $D'' = (-1,1)$

- zeri f'' $\begin{cases} x(2x^2-3) = 0 \rightsquigarrow x = 0, \pm\sqrt{\frac{3}{2}} \\ -1 < x < 1 \end{cases}$

$\Longrightarrow \pm\sqrt{\frac{3}{2}}$ fuori dal dominio

FLESSO IN $x = 0 \rightsquigarrow f'(0) = 1$

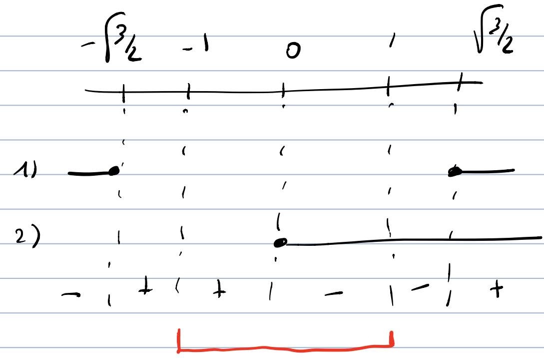


- segno $f'' > 0$

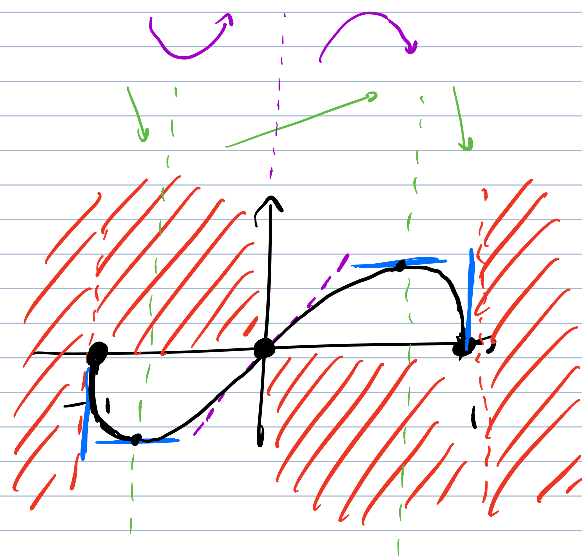
$$\frac{(2x^2-3)x}{(1-x^2)^{3/2}} > 0 \underset{\text{nel dominio}}{\rightsquigarrow} (2x^2-3)x > 0$$

1) $2x^2 - 3 > 0$ $\quad -\sqrt{3/2} \quad \sqrt{3/2} \quad$ $x < -\sqrt{\frac{3}{2}} \vee x > \sqrt{\frac{3}{2}}$

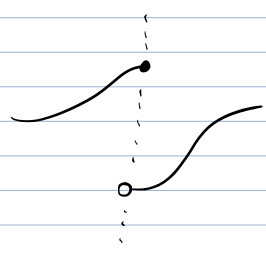
2) $x > 0$



$$f'' > 0 \quad \text{se} \quad -1 < x < 0$$



RIPASSO DISCONTINUITÀ



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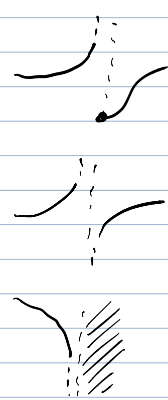
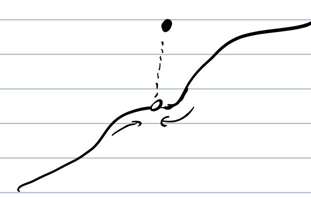


II



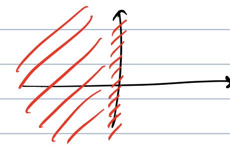
III

551 $f(x) = x^2 \log(x)$

(escluso
[incluso

- dominio: $x > 0$ (log) $\qquad D = (0, +\infty)$

- parità: dominio non è simmetrico
  ↳ né pari, né dispari

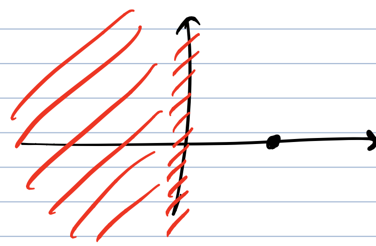


- intersezioni assi: Y | non ne ho a causa del dominio

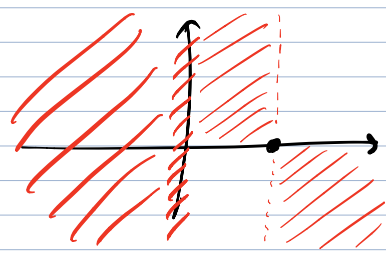
X | $\begin{cases} x^2 \log(x) = 0 \\ x > 0 \end{cases} \rightarrow x^2 = 0 \ \vee \ \log(x) = 0$

$x^2 = 0 \rightarrow x = 0$ fuori dal dominio

$\log(x) = 0 \rightarrow \underline{x = 1}$



- segno $\begin{cases} x^2 \log x > 0 \\ x > 0 \end{cases} \xrightarrow[\text{dominio}]{} \log x > 0 \rightsquigarrow x > 1$



- bordi del dominio $x \to 0^+$, $x \to +\infty$

0^+ | $\lim_{x \to 0^+} x^2 \log x = 0 \cdot (-\infty) \leftarrow$ FORMA INDETERMINATA

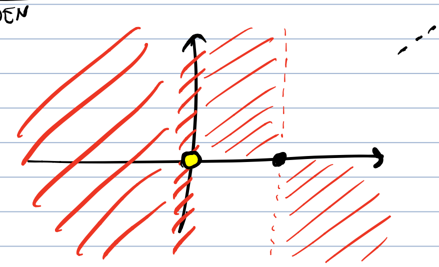
1) gerarchia zeri/infiniti $\quad \log \ll x^\alpha \ll b^x$

2) teo Hopital

$$\lim_{x \to 0^+} \frac{\log x}{1/x^2} = \frac{\infty}{\infty}$$

$$\lim_{x\to 0^+} \frac{\log x}{1/x^2} \overset{H}{=} \lim_{x\to 0^+} \frac{\overbrace{1/x}^{\text{derivo NUM}}}{\underbrace{-2\frac{1}{x^3}}_{\text{derivo DEN}}} = \lim_{x\to 0} \left(-\frac{1}{2}\right) x^2 = 0$$

$$\lim_{x\to 0^+} f(x) = 0$$



$\underline{+\infty|}$ $\lim_{x\to +\infty} x^2 \log x = +\infty$

• derivata prima $f'(x) = 2x \log x + x^2 \frac{1}{x} =$

$= 2x \log x + x = x(2\log x + 1)$

- dominio $D' = D = \{x > 0\}$

- bordo dominio $\underline{0^+|}$ $\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} x(2\log x+1) = 0$

$\underline{+\infty|}$ $\lim_{x\to +\infty} f'(x) = +\infty$ NO ASI OBLIQUO

- pendenza nello zero $x=1$, $f'(1) = 1 \cdot (2 \cdot 0 + 1) = 1$

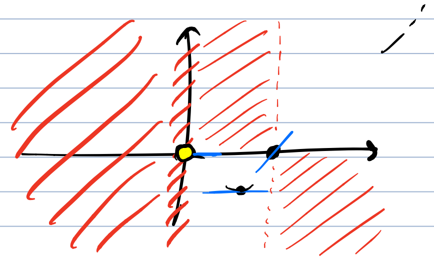
- zeri $\begin{cases} x(2\log x + 1) = 0 \\ x > 0 \end{cases} \longrightarrow x = 0$ oppure

$2\log x + 1 = 0$

$\log x = -\frac{1}{2}$

$e^{-1/2} = x$

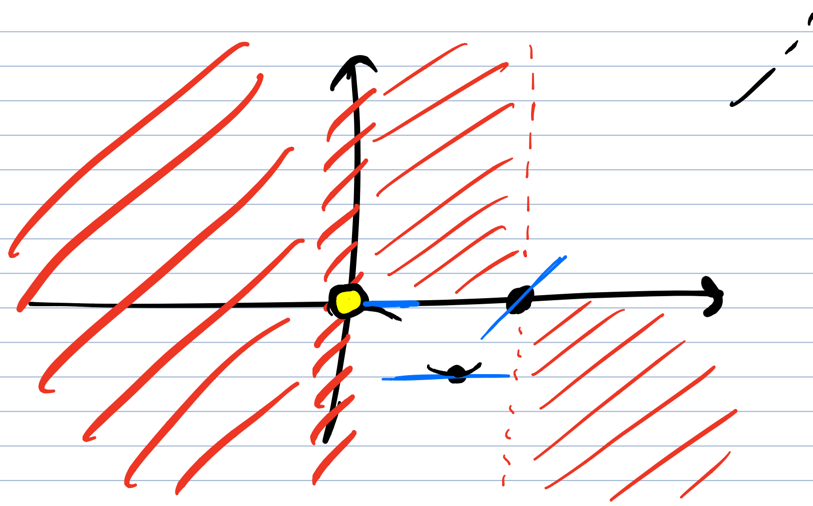
$x = \frac{1}{\sqrt{e}} > 0$



pto stazionario in $\left(\frac{1}{\sqrt{e}} ; -\frac{1}{2e}\right)$

$$f\left(\frac{1}{\sqrt{e}}\right) = \frac{1}{e} \log(e^{-1/2}) \underset{\uparrow}{=} -\frac{1}{2e} \log e = -\frac{1}{2e}$$

PROP LOG

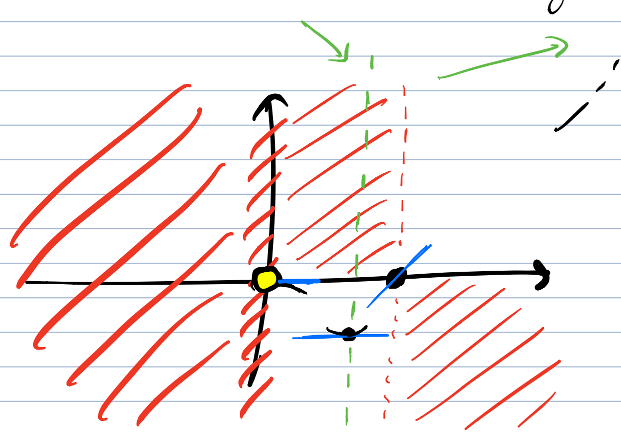


- segno $f'(x) = \begin{cases} x\,(2\log x + 1) > 0 \\ x > 0 \end{cases}$

$\underset{\text{dominio}}{\rightsquigarrow}$ $2\log x + 1 > 0$

$\log x > -1/2 \rightsquigarrow x > \frac{1}{\sqrt{e}}$

log monotono crescente

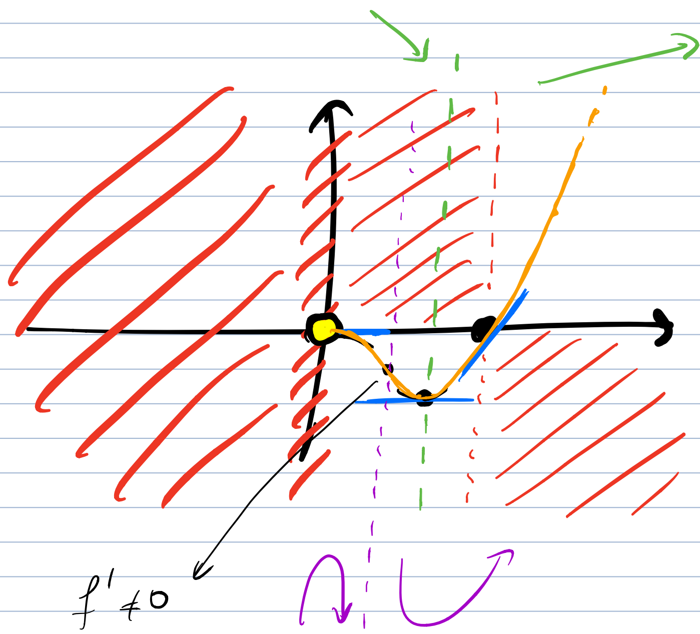


$f'(x) = x(2\log x + 1)$

$$- \quad f''(x) = 2\log x + 1 + x\left(2\frac{1}{x} + 0\right) = 2\log x + 1 + 2 = 2\log x + 3$$

zeri $f''(x)$ (flessi) $\qquad 2\log x + 3 = 0 \qquad \log x = -\frac{3}{2} \qquad x = e^{-3/2}$

$$f(e^{-3/2}) = e^{-3}\log e^{-3/2} = -\frac{3}{2e^3}$$



segno di $f''(x) \qquad 2\log x + 3 > 0 \xrightarrow{\text{come sopra}} x > e^{-3/2}$