MAIL: VITTORIO.ERBA@UNIMI.IT (DOTTORANDO UNIMI)

- HTTPS://VITTORIOERBA.GITHUB.IO/TEACHING_PRECORSO2021.HTML
  - REGISTRAZIONI
  - CANALE PDF
  - PROGRAMMA
  - MATERIALE PER ESERCIZI/TEORIA PDF → 2 ESERCIZIARI; → LIBRO TESTO OPENSOURCE

- PAUSE 9:00 - 9:45 (AULA A)
  10:00 - 10:45
  11:00 - 11:45

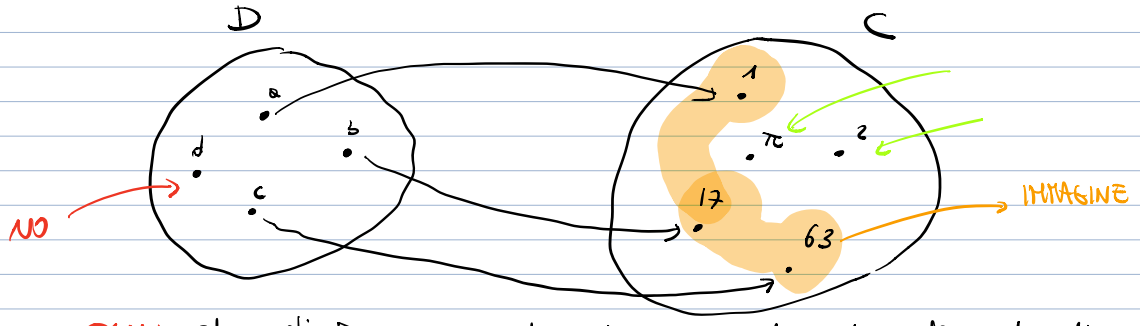
- PROGRAMMA 1) FUNZIONI 2) DERIVATE 3) STUDI DI FUNZIONE 4) INTEGRALI 5) VETTORI

LEZIONE 1: FUNZIONI

Definizione: D) dominio (insieme) → INPUT DELLA FUNZIONE

C) codominio (insieme) → OUTPUT DELLA FUNZIONE

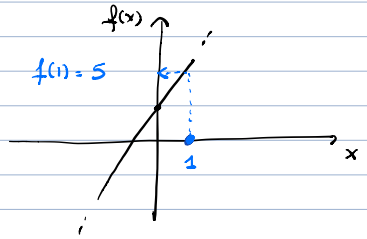
f) assegnazione da D a C (ogni elemento di D viene mappato verso un elemento di C)



OGNI elem. di D assegnato ad 1 e 1 solo elemento di C

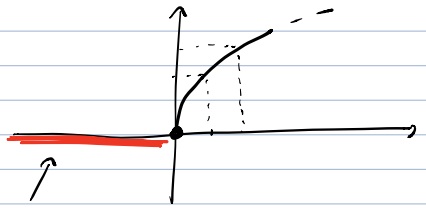
es: $f: \mathbb{R} \to \mathbb{R}$ $f(x) = 2x+3$

DOMINIO CODOMINIO



es: $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \sqrt{x}$

NON È UNA FUNZIONE



f si "rompe"

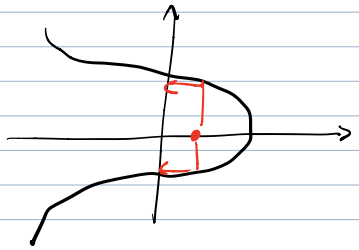
COMPRESO ESCLUSO

$f : [0, +\infty) \to \mathbb{R}$ $\quad f(x) = \sqrt{x}$

È UNA FUNZIONE

$\mathbb{R}^+ = [0, +\infty)$ IMMAGINE $[0, +\infty)$???

es: $f : \mathbb{R} \to \mathbb{R}$

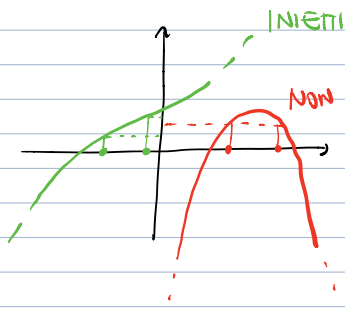


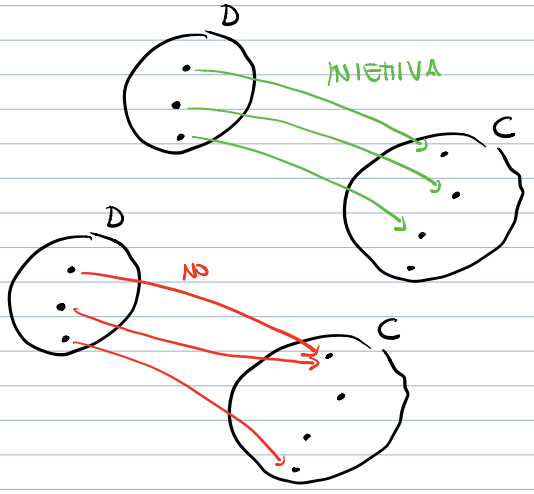
NON È FUNZIONE

INIETTIVA / SURIETTIVA / BIUNIVOCA

1) INIETTIVA se $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$

$(x_1, x_2 \in D)$ $\quad (f(x_1), f(x_2) \in C)$

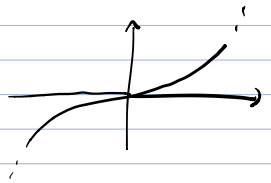




MOTIVAZIONE expr 1 ≠ expr 2

$\Rightarrow f(\text{expr 1}) \neq f(\text{expr 2})$

se f iniettiva



$4x+5 \neq y^7 - z + \sqrt{x}$

$(4x+5)^3 \neq (y^7 - z + \sqrt{x})^3$

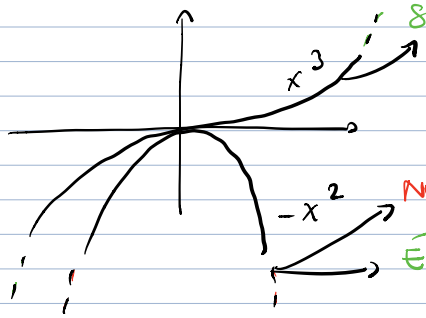
$f: \mathbb{R} \to \mathbb{R}$

$f(x) = x^3$

2) SURIETTIVA $\forall y \in C \quad \exists x \in D : f(x) = y$

$\forall y \in C$ esiste una PREIMMAGINE di y

(INPUT che ha come OUTPUT y)

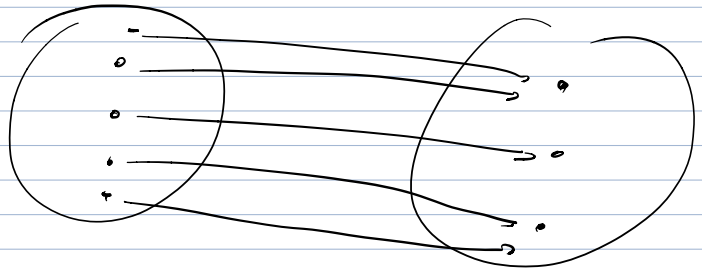


SURIETTIVA $(C = \mathbb{R})$

NON È SURIETTIVA $C = \mathbb{R}$

È SURIETTIVA SE $C = (-\infty, 0]$

SURIETTIVA $\Leftrightarrow$ CODOMINIO = IMMAGINE



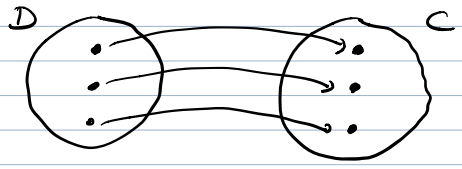
3) BIUNIVOCA = INIETTIVO + SURIETTIVO

idea: - la potete invertire

- esiste $f^{-1}(x)$

$\forall y \in C \quad \exists! \quad x \in D \quad t.c. \quad f(x) = y$

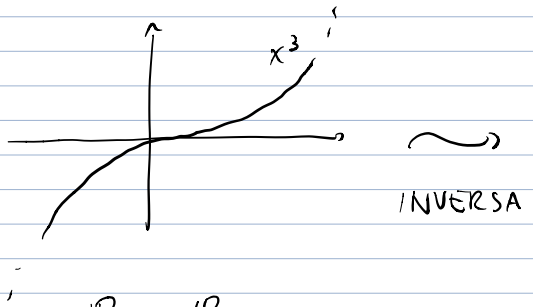
ESISTE UNICO

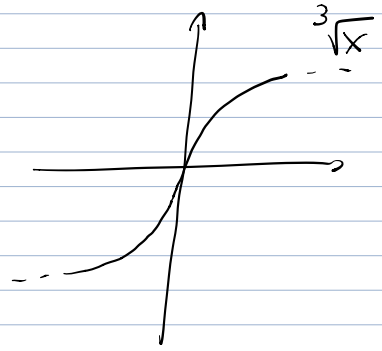


INIETTIVA? SÌ

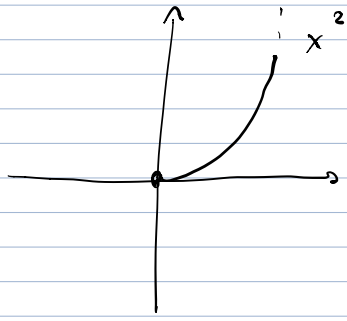
SURIETTIVA? SÌ

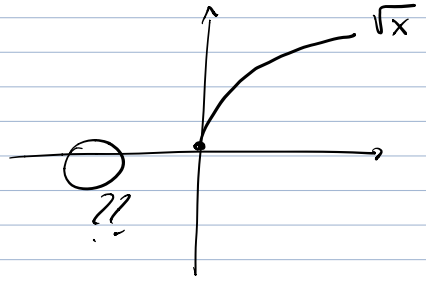
⇒ BIUNIVOCA





$\mathbb{R} \to \mathbb{R}$ $\quad$ $\mathbb{R} \to \mathbb{R}$





$[0, +\infty) \to \mathbb{R}$ $\quad$ $\mathbb{R} \to [0, +\infty)$

1)

INIETTIVA

MA NON SURIETTIVA

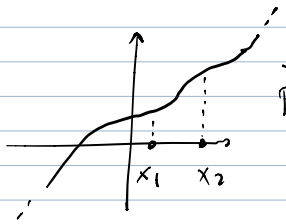
NON BEN DEF

2) $[0,+\infty) \to [0,+\infty)$ — INIETTIVA + SURIETTIVA

$[0,+\infty) \to [0,+\infty)$ — OK !

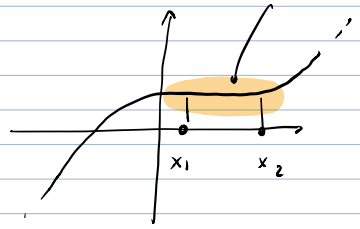
CASO PARTICOLARE: FUNZIONI REALI $f(x) = y \quad x, y \in \mathbb{R}$

1) MONOTONIA



NO zone piatte

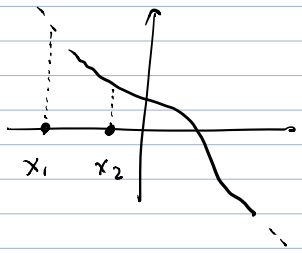
CRESCENTE



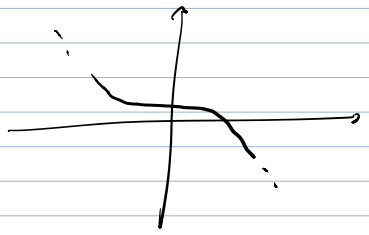
zone piatte

CRESCENTE IN SENSO LATO

$\forall x_1, x_2 \in D : x_2 > x_1 \Rightarrow f(x_2) \geq f(x_1)$



DECRESCENTE

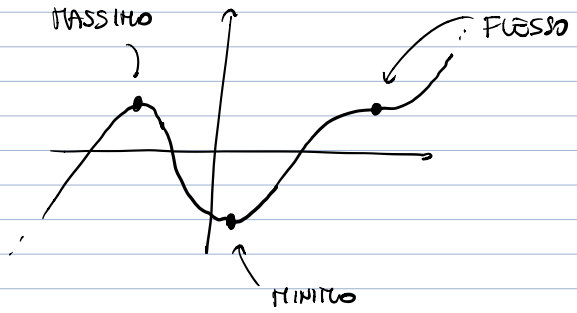


DECRESCENTE IN SENSO LATO

$\forall x_1, x_2 \in D : x_2 > x_1 \Rightarrow f(x_2) \leq f(x_1)$

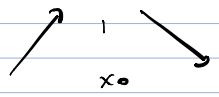
MOTIVAZIONE: se f è crescente/decrescente (non in senso lato) $\Rightarrow$ iniettiva

2) PUNTI STAZIONARI, MASSIMI, MINIMI

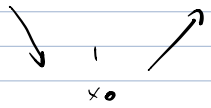


NOZIONI LOCALI

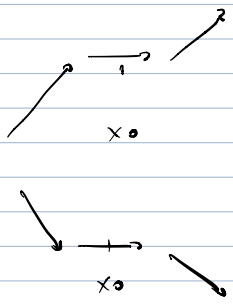
MAX: punto x_0 t.c. in un suo intorno piccolo a piacere
$f(x)$ è crescente se $x < x_0$
$f(x)$ è decrescente se $x > x_0$

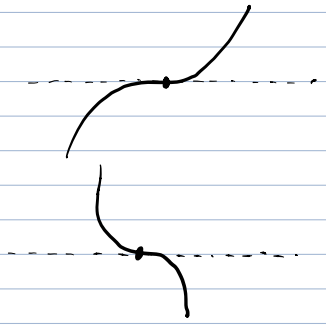


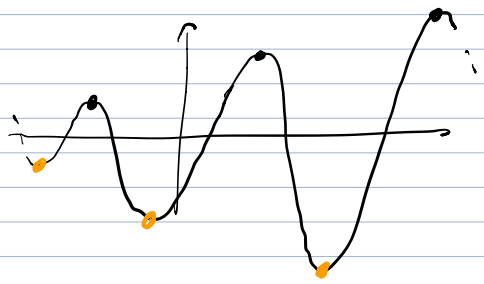
MIN:



FLESSO



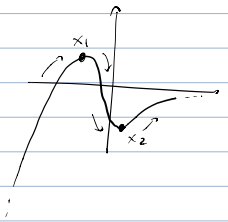




- MAX LOCALI
- MIN LOCALI

esiste MAX/MIN GLOBALE?

MAX GLOBALE: più alto di f



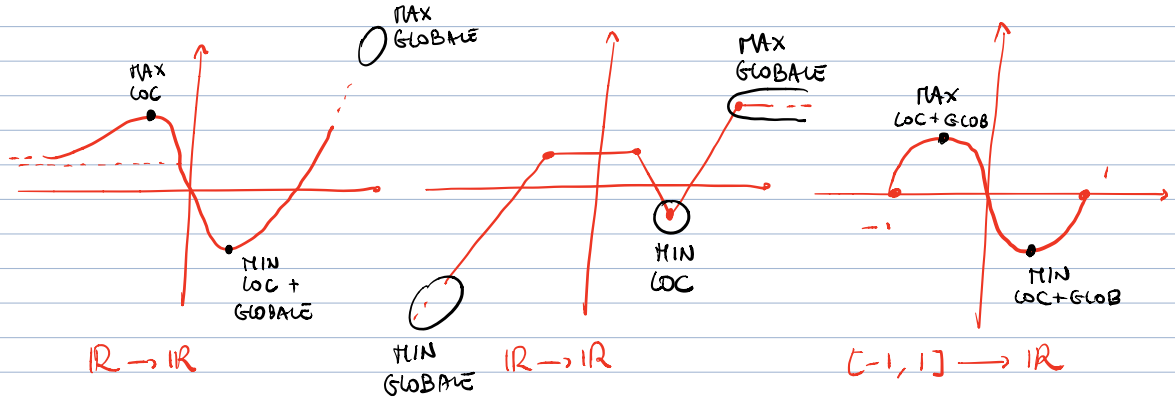
x_1 : MAX LOCALE e GLOBALE

x_2 : MIN LOCALE

MIN GLOBALE $x = -\infty$

$f(x) = -\infty$

ESERCIZIO

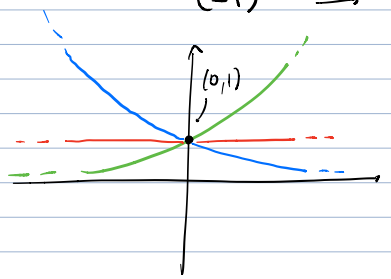


- GRAFICI / DOMINIO / PROPRIETÀ

  - esponenziali $f(x) = a^x$ $a \in \mathbb{R}$, $a > 0$

$(-1)^{\pi}$ come scelgo il segno? Non lo so fare

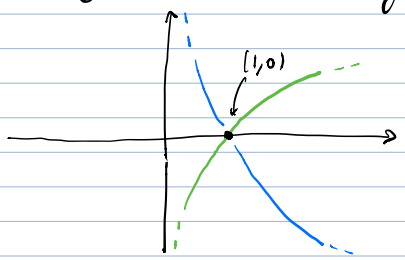
$(-1)^{2/7} \to \sqrt[7]{(-1)^2} = \sqrt[7]{1} = 1$



$a > 1$
$0 < a < 1$
$a = 1$

dominio $\mathbb{R}$
codominio $\mathbb{R}$ o lo restringiamo a $(0, +\infty)$
f è monotona crescente $a > 1$
" decrescente $0 < a < 1$
f è biunivoca $(\mathbb{R} \to (0, +\infty))$ se $a \neq 1$

  - logaritmo $f(x) = \log_a x$ $a \in \mathbb{R}$, $a > 0$, $a \neq 1$



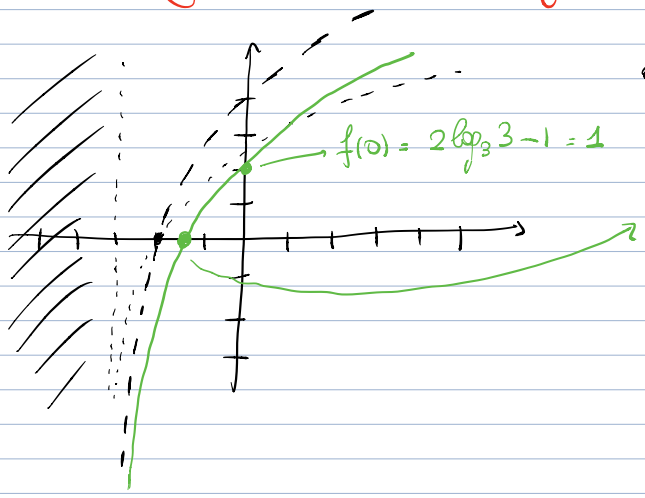
$a > 1$
$0 < a < 1$

dominio $(0, +\infty)$, $x > 0$
codominio $\mathbb{R}$
f è monotona crescente $a > 1$
" decrescente $0 < a < 1$
f è biunivoca $((0, +\infty) \to \mathbb{R})$

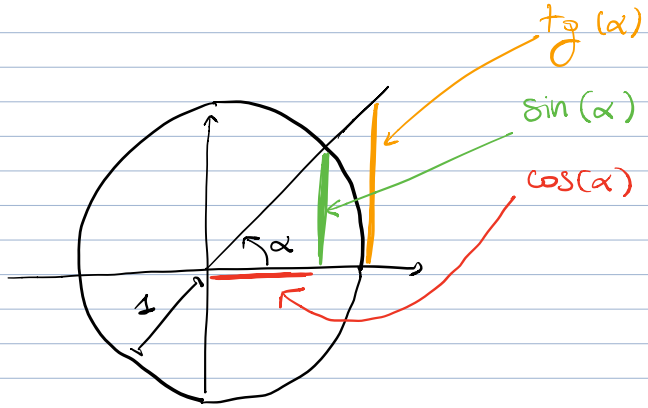
COMPITI : LIBRO MATE SITO VOLUME 4 pg 45

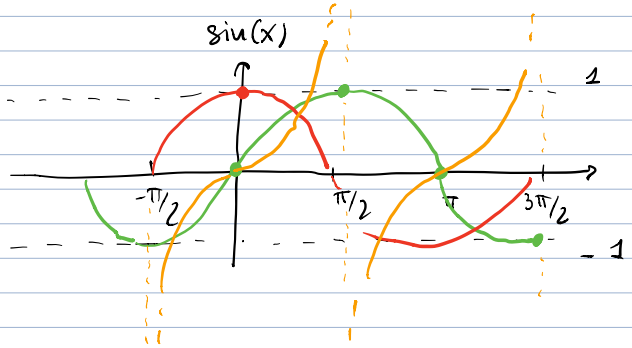
- SEMPLIFICAZIONI
- EQUAZIONI / DISEQUAZIONI

ESERCIZIO : grafico di $f(x) = 2\log_3(x+3) - 1$



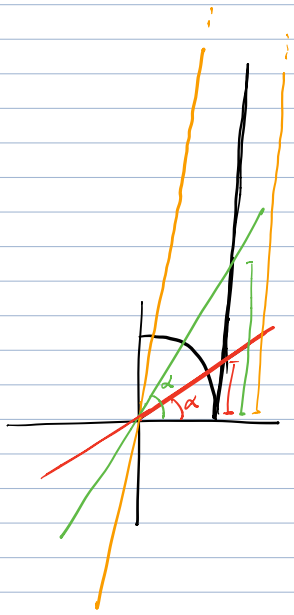
- funzioni trigonometriche





TAN / TG
periodico π
non iniettiva
dominio
$x \neq \frac{\pi}{2} + K\pi$
K intero
codominio $\mathbb{R}$

SIN COS
periodico 2π
non iniettiva
dominio $\mathbb{R}$
codominio $[-1, 1]$



$tg(\alpha) = 0.7$

$tg(\alpha) = 2$

$tg(\alpha) \longrightarrow +\infty$

$tg(x) \xrightarrow[x \to \pi/2]{} \pm\infty$

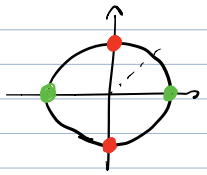
$x = \frac{\pi}{2} \overset{tg}{\longmapsto} \pm\infty \notin \mathbb{R}$

$\Rightarrow$ dominio deve escludere $x = \frac{\pi}{2} + k\pi$ (k intero)

→ TEORIA VOLUME 4 SEZIONE 7

↳ esercizi: - semplificazioni, regole addizione ecc...

- equazioni / disequazioni
- grafici



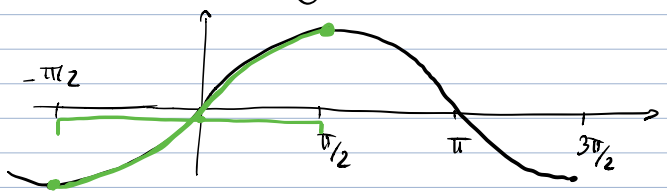
dove $\sin / \cos / tg = 0$

$\sin(x) = 0 \quad x = 0, \pi + 2k\pi = 0 + k\pi$

$\cos(x) = 0 \quad x = \frac{\pi}{2}, \frac{3\pi}{2} + 2k\pi = \frac{\pi}{2} + k\pi$

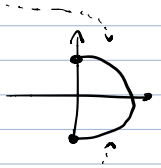
$tg(x) = 0 \quad x = 0 + k\pi$

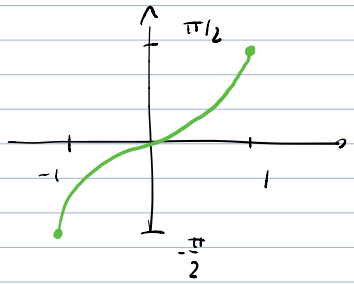
- funzioni trigonometriche inverse



$\sin(x) : [-\frac{\pi}{2}, \frac{\pi}{2}] \longrightarrow [-1, 1]$

è biunivoca

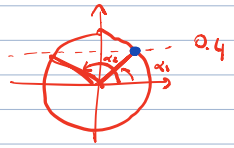




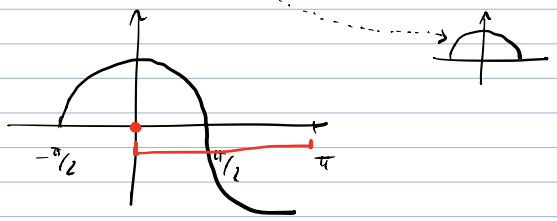
$\arcsin(x) : [-1, 1] \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

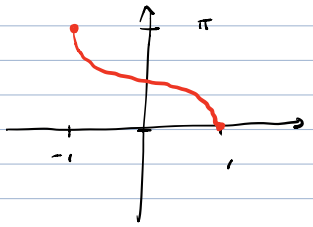
$\sin(x) = 0.4$

ma $\arcsin(0.4) = \alpha_1$

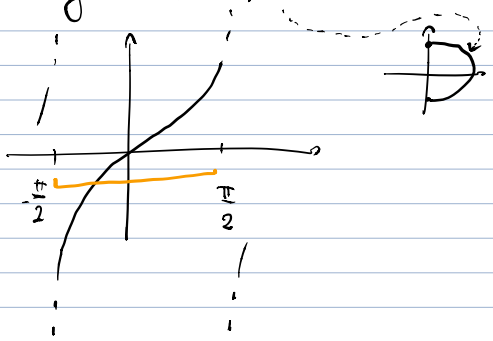


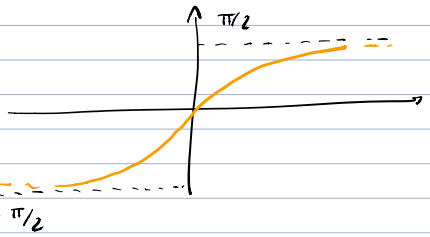
$\cos(x) : [0, \pi] \longrightarrow [-1, 1]$ e inversa è $\arccos(x) : [-1, 1] \to [0, \pi]$





$\text{tg}(x) : (-\pi/2, \pi/2) \longrightarrow \mathbb{R}$ inversa $\arctg(x) : \mathbb{R} \longrightarrow (-\pi/2, \pi/2)$





COMPITI: RACCOLTA ES SU CALCOLO DEL DOMINIO (SITO)