$\hat{O}: \{a_n\}_{n=1}^{+\infty} \in \ell^2(\mathbb{C}) \longmapsto \left\{\frac{a_n}{\sqrt{n(n+1)}}\right\}_{n=1}^{+\infty} \in \ell^2(\mathbb{C})$

$\|\hat{O}\|_{op}$, etc. ...

Chebyshev: $T_0(x)=1$, $T_1(x)=x$

$$T_{n+1}(x) = 2x\,T_n(x) - T_{n-1}(x)$$

da $2\cos(n\theta)\cos\theta = \cos((n+1)\theta) + \cos((n-1)\theta)$

$$x_k = \cos\left(\frac{\pi(k+1/2)}{n}\right), \quad T_n(1) = 1$$
$$T_n(-1) = (-1)^n$$

$$h_0 = \pi, \quad h_{n>0} = \frac{\pi}{2}$$

Approx. $\sin\theta\, e^{\cos(\theta)}$ con pol di grado 3

in $\cos(\theta)$ in $\theta \in [0, \pi)$

$$\int_0^\pi \left| \sin\theta\, e^{\cos\theta} - a\cos^3\theta - b\cos^2\theta - c\cos\theta - d \right|^2 d\theta$$

$\cos\theta = x \qquad d\theta = \frac{-dx}{\sqrt{1-x^2}}$

$$\int_{-1}^{1} \frac{\left|\sqrt{1-x^2}\, e^{x} - ax^3 - bx^2 - cx - d\right|^2}{\sqrt{1-x^2}}\, dx \to \text{Chebyshev}$$

$$C_0 = \frac{e^2 - e}{\pi e} \qquad C_1 = \frac{4}{\pi e}$$

$$C_2 = \frac{2}{\pi e}\left(e^2 - 9e\right) \qquad C_3 = -\frac{2}{\pi e}\left(8e^2 - 58e\right)$$